Manipulated News : Electoral Competition and Mass Media *

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Abstract

This paper studies a Downsian voting model including mass media, and characterizes how the mass media affect outcomes in electoral competitions. The media can observe the proposed policies by the two candidates, but the voter cannot. The media then send news about the proposed policies before voting occurs. We assume that information regarding the proposed policies is verifiable; that is, the media cannot fabricate the information, but can withhold it. In the model with single medium, we show that the voter's decision making could be incorrect ex post in any equilibrium when the medium's preference is sufficiently different from that of the voter. Appealing to the voter then becomes less attractive to the candidates. Furthermore, the candidates have an incentive to influence the medium's behavior through policy settings. Through the distortions in the behaviors of the voter and the candidates, the equilibrium outcomes are distorted in favor of the medium; that is, the median voter theorem could fail. This distortion mechanism can be observed even if there are multiple media when their preferences are like biased. However, if the media's preferences are opposing biased, then the median voter theorem holds.

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Key Words: voting, mass media, hard information, influence incentive, median voter theorem

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1 Introduction

This paper considers the question of how mass media affect electoral competitions. We regard the mass media as information providers, and discuss what happens in elections if the information providers can manipulate information about the proposed policies that is released to the voter. In reality, the mass media report several types of information regarding elections, and their reports influence both candidates and voters. For example, the mass media frequently report who is a candidate, what his/her proposed policy is, and to what extent the policy is endorsed by voters. That is, the interactions between candidates and voters in real-world elections are indirect in the sense that the mass media exist between the candidates and the voters, and the mass media provide essential information for the decision makings of the candidates and the voter. Most voters use the news as an information source for voting instead of directly observing the information. Candidates may also use mass media outlets to change their policies if they are little endorsed by voters. There is then no doubt that the mass media has a substantial influence on political outcomes.

However, in the literature of voting theory, the role of the mass media is conspicuously omitted. Most standard voting models represent the interaction between candidates and voters as direct in the sense that the players are the candidates and the voters; the voters can directly observe the proposed policies, and the candidates can obtain feedback directly from the voters. If the mass media always provide enough information for our correct decision makings, then the setup used in the literature is a good approximation of the real world. However, it is well reported that we frequently observe media bias in media outlets. That is, reports from the mass media is not raw information but manipulated; for example, unbalanced reports, fabrications of facts and hiding of fact. One interpretation of this phenomenon is that the media bias reflects the preferences of journalists or the owners of the media who have a big impact on contents of news. In other words, the media outlets are slanted as a consequence of optimizations of the owners or the journalists. If we understand the media bias with this interpretation, we could easily imagine that the mass media potentially have an incentive to manipulate information released to the voter when their preferences on the issue of an election are different from those of the majority voters. The aim of this paper is making clear the mechanism of how the outcomes in the election is affected if there exist mass media that potentially manipulate the information released to the voter.

Because of this motivation, we construct a simple Downsian voting model including mass media that can manipulate information about proposed policies. In the model, there exist two candidates, single/multiple media and one voter. Unlike standard models, the voter cannot directly observe the policies proposed by the candidates. Instead, the voter learns this information through reports from the media. In other words, we consider the following two-stage game. In the first stage, the two candidates simultaneously propose policies that only the media can observe. In the second stage, the media sends news regarding those policies after which the voter chooses one of the candidates. We assume that the information about the proposed policies is hard information; that is, it is verifiable by either a third party or the voter following the election. Hence, the second stage is represented by a persuasion game in the literature of strategic communication. That is, the media can withhold unfavorable information about policies, but cannot fabricate the information. Under this environment, we mainly pursue the following two questions. First, how much relevant information do the media disclose? Second, how does the existence of the media distort electoral outcomes as compared with the no-media case?

The results are as follows. In the model with single medium, we show that the voter's decision making could be incorrect ex post in any equilibrium when the preferences between the medium and the voter sufficiently diverge. Because of the voter's ex post incorrect decision making, appealing to the voter becomes less attractive to the candidates. The candidates then have an incentive to win the election by influence the medium's behavior through policy settings instead of appealing to the voter. As a result, the equilibrium outcomes are distorted in favor of the medium when compared with no media model through the distortions in the voter's and the candidates's behaviors; that is, the median voter theorem could fail. This distortion mechanism could observe in the model with multiple mass media. If there exist two media and their preferences are like biased, then the same distortion mechanism can be observed. However, if the media's preferences are opposing biased, then no information distortion occurs. That is, because of the monitoring by the media, the median voter theorem holds again under this setup.

This paper is organized as follows. In the following subsection, we briefly review the related literature. Through Sections 2 to 4, we focus on the model with single medium. Section 2 defines the formal model. In Section 3, we analyze a benchmark model without mass media, and consider a model with single medium in Section 4. In Section 5, we consider the model with multiple media, and conclude the paper in Section 6.

1.1 Related literature

This paper is based on several branches of economics. First, this paper is positioned in the literature of political economy as a paper that demonstrates non-policy-convergence results by changing the basic setup. As the basic framework of the analysis, we adopt the Downsian voting model in which the candidates are fully office motivated, as introduced by Downs (1957). In this environment, the equilibrium policies converge to the median voter's ideal policy. This is the well-known *median voter theorem*. As Roemer (2001) explains, because the policy convergence is inconsistent with observations in the real world, filling the gap between the model predictions and these observations is one of the main concerns in the literature. Included in this branch of work are several papers deriving policy divergence results by modifying the setup. Calvert (1985) derives the divergence result by assuming that the candidates are policy motivated and face uncertainty about the location of the median voter. Palfrey (1984) and Kartik and McAfee (2007) also derive the divergence results by introducing the entrant of the third candidate and the voters' preferences on the candidates' characters, respectively.

In this paper we consider the situation where the voter faces uncertainty. In this direction, Kikuchi (2010) derives the policy divergence results by assuming that voter faces uncertainty about the state of nature that is soft private information of the candidates. Particularly, we focus on the situation where the voter has imperfect information about proposed policies like McKelvey and Ordeshook (1985). McKelvey and Ordeshook (1985) derive the policy convergence result because the uninformed voters can partially learn the information about the proposed policies by observing the poll which reflect the behaviors of the informed voters. While the information released to the voter is exogenously fixed in McKelvey and Ordeshook (1985), the mass media strategically selects the information released to the voter in this paper. The limited information setup then derives the divergence result.

Second, this paper is also related to the economics of the mass media. According to the recent survey in Prat and Strömberg (2011), this field includes the following branches of theoretical research; (i) the analysis of media capture by the government, (ii) the analysis of free media with the question of "What do media cover?" and (iii) the question of "How do media cover?" Of these, Besley and Prat (2006) is positioned in the first branch by showing that the number of mass media in the market is important for media capture by the government. As an example of work in the second branch, Strömberg (2004) examines the effect of topic selection on public policy, showing that because the mass media provide valuable news to the consumers that are likely to buy a particular newspaper, the public policy is distorted in favor of these consumers.

In the final branch of mass media economics, Mullainathan and Shleifer (2005), Baron (2006) and Gentzkow and Shapiro (2006) focus on biased media outlets and the reason why they exist. In this regard, Mullainathan and Shleifer (2005) emphasize the demand side; that is, media outlets are biased to satisfy the demand of consumers who want to read articles consistent with their own beliefs. Conversely, Baron (2006) and Gentzkow and Shapiro (2006) emphasize the supply side.

Baron (2006) argues that biased outlets are the consequence of the career concerns of journalists and the cost-minimizing behavior of media owners. In Gentzkow and Shapiro (2006), media outlets are biased toward the prior beliefs of consumers in order to build a good reputation for the quality of news. This paper is also located in this branch of work, in which Chan and Suen (2008, 2009) employ similar models, in that we analyze the consequences of the mass media for electoral competitions. Chan and Suen (2008, 2009) regard the mass media as "watchdogs" of elections, which provide subjective endorsements to the voters when the voter cannot distinguish which candidate is better after observing hard information about the proposed policies by the candidates. That is, Chan ans Suen (2008, 2009) implicitly assume that the media never manipulate the hard information. This paper revisits this assumption by analyzing the model in which the mass media can manipulate how much hard information is released to the voter.

Third, we use a persuasion game framework to describe the media manipulation of hard information. Persuasion games are sender–receiver games with hard private information, as first formalized by Milgrom (1981), for which there is now a voluminous literature, for example, Milgrom and Roberts (1986), Seidmann and Winter (1997) and Giovannoni and Seidmann (2007). In contrast with cheap-talk games, like Crawford and Sobel (1982), the sender's private information in this framework is verifiable, so the information cannot be misreported, but the sender can withhold unfavorable information. Our analysis thus is based on a similar general persuasion game to Miura (2011). Furthermore, from the viewpoint of this literature, this paper analyzes a hierarchical persuasion game in which the sender's private information is affected by the others' strategies.

2 The Model

There are four players in our model: candidates 1 and 2, one medium and one (median) voter.[1] They play the following two stage game. In the first stage, called the *policy setting stage*, each candidate simultaneously proposes a policy, and only the medium can observe the proposed policies. In the second stage, called the *information disclosure stage*, the medium sends a message about the proposed policies to the voter. After observing the message, the voter casts the ballot for one of the candidates. The winning candidate then implements the proposed policy.

Let $X \equiv \{l, 0, r\} \subset \mathbb{R}$ be the set of available policies for the candidates with $l < 0 < r$ and $|l| > |r|$. Let $x_i \in X$ be the policy proposed by candidate $i \in \{1, 2\}$, and $x \equiv (x_1, x_2) \in X^2 \subset \mathbb{R}^2$

[1]We define the model with single medium here. The model with multiple media is defined in Section 5, which can be easily defined as an analogy of the single medium model. Throughout the paper, we treat the candidates and the voter as male and the media as female.

describe a pair of the proposed policies by the candidates. We assume that the information regarding x is *hard information*; that is, verifiable information. In addition, we assume that the medium can correctly observe x, but the voter cannot. Hence, the information about x is the medium's private information at the information disclosure stage.[2]

Let $M(x) \equiv \{x, \phi\}$ be the message space of the medium when she observes policy pair x in the policy setting stage. The element x represents the medium's disclosure behavior. That is, the medium tells the voter what she observes. On the other hand, the element ϕ represents the withholding of information by the medium. That is, the medium completely conceals what she observes and tells nothing to the voter.[3] Note that the medium cannot say that the observed policy pair is x' when she observes $x \neq x'$ because the information is verifiable.[4] Let $M \equiv \cup_{x \in X^2} M(x)$ be the universal message space, and $m \in M$ be the generic notation of the medium's message. Let $Y \equiv \{y_1, y_2\}$ be the voter's action space, where y_i represents that the voter casts the ballot for candidate $i \in \{1, 2\}$, and $y \in Y$ describes the generic notation of the voter's action.

We assume that there are two types of candidates: a *non-policy type* and a *policy type*. The non-policy type is the standard office-motivated strategic type of candidate. On the other hand, the policy type is a behavioral type of candidate that always proposes his preferred policy. We assume that if candidate 1 (resp. 2) is the policy type, then he always proposes policy r (resp. l). That is, we assume an asymmetry between the candidate.[5] Let $\Theta \equiv \{\theta_N, \theta_P\}$ be the candidates' type space, and θ_N (resp. θ_P) represent the non-policy (resp. policy) type. We assume that candidate i's type $\theta_i \in \Theta$ is candidate i's private information, and θ_1 and θ_2 are independently determined. Let $p > 0$ be the probability that each candidate is the non-policy type, and assume this is common knowledge.

The players' preferences are defined as follows. Define the non-policy-type candidate i's von Neumann–Morgenstern utility function $u_i : Y \to \mathbb{R}$ by:

$$u_i(y) \equiv \begin{cases} 1 & \text{if } y = y_i \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

We assume that the medium and the voter have single-peaked preferences. Define the voter's von

[2]These assumptions seem reasonable because in the real world this information is explicitly included in the manifesto for each candidate, which anyone can check if they wish. However, the majority of voters learn the information through reports in the mass media; they seldom check the information themselves.

[3]We can easily extend the model to allow the medium to partially disclose the information about x. However, we obtain similar results. Accordingly, to simplify the analysis, we restrict the medium's message space as above.

[4]In other words, we implicitly assume that the medium bears huge costs for misreporting, for example, her bad reputation is widely known to people because of the verifiability of the information.

[5]We discuss this assumption in Appendix B.

Neumann–Morgenstern utility function $v : X^2 \times Y \to \mathbb{R}$ by:

$$v(x, y) \equiv \begin{cases} -|x_1| & \text{if } y = y_1 \\ -|x_2| & \text{if } y = y_2. \end{cases} \tag{2}$$

Similarly, define the medium's von Neumann–Morgenstern utility function $w : X^2 \times Y \times \mathbb{R} \to \mathbb{R}$ by:

$$w(x, y, b) \equiv \begin{cases} -|x_1 - b| & \text{if } y = y_1 \\ -|x_2 - b| & \text{if } y = y_2. \end{cases} \tag{3}$$

The voter's ideal policy is 0, but that of the medium is $b > 0$. Hence, the parameter b represents the difference between the preferences of the voter and the medium. We refer to this parameter throughout the paper as *media bias*. We assume that the level of media bias is common knowledge.

The timing of the game is formalized as follows. At the policy-setting stage, nature chooses candidate i's type $\theta_i \in \Theta$ according to the prior distribution p, and only candidate i correctly learns his own type θ_i. Then, given θ_i, each candidate simultaneously proposes a policy $x_i \in X$. Only the medium can correctly observe the pair of proposed policies $x \in X^2$. At the information disclosure stage, given the observed pair x, the medium sends a message $m \in M(x)$. After observing the message, the voter undertakes an action $y \in Y$. The policy announced by the winning candidate is then implemented.

The players' strategies are defined as follows. The non-policy-type candidate i's strategy $\alpha_i \in \Delta(X)$ is a probability distribution over the policy space for $i \in \{1, 2\}$. This is represented by $\alpha_i = (\alpha_i^0, \alpha_i^r, 1 - \alpha_i^0 - \alpha_i^r)$ where α_i^j represents the probability that candidate i of the non-policy type proposes policy j. With some abuse of notation, a pure strategy of the non-policy-type candidate i is simply described as $\alpha_i = x_i$. The medium's strategy $\beta : X^2 \to M$ is a function from an observed policy pair to a message. The voter's strategy $\gamma : M \to \Delta(Y)$ is a function from an observed message to a probability distribution over the voter's action set Y. The voter's strategy is represented by $\gamma(m) = (q(m), 1 - q(m))$, where $q(m)$ represents the probability that the voter casts the ballot for candidate 1 when he observes message m. With further abuse of notation, the voter's pure strategy is simply represented by $\gamma(m) = y$. Let $\mathcal{P} : M \to \Delta(X^2)$ represent the voter's posterior belief, which is a function from an observed message to a probability distribution over the set of possible policy pairs X^2.

We use the perfect Bayesian equilibrium (hereafter, PBE) as a solution concept. Because the voter knows that only the medium that observes policy pair x' can send message $m = x'$, we insert the following requirement as a restriction to off-equilibrium-path beliefs.

Requirement 1 *For any $x' \in X^2$, if the voter observes a message $m = x'$, then the voter's posterior belief satisfies $\mathcal{P}(x = x'|m = x') = 1$.*

Definition 1 ***Perfect Bayesian Equilibrium***
A quintuple $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE if it satisfies the following conditions;

(i) *For every $x_i \in supp(\alpha_i^*)$ and $i, j \in \{1, 2\}$ with $j \neq i$, $x_i \in \arg\max_{x_i' \in X} \mathbb{E}[u_i(\gamma^*(\beta^*(x_i', \alpha_j^*)))]$;*

(ii) *For every $x \in X^2$, $\beta^*(x) \in \arg\max_{m \in M(x)} w(x, \gamma^*(m), b)$;*

(iii) *For every $m \in M$ and $y \in supp(\gamma^*(m))$, $y \in \arg\max_{y' \in Y} \mathbb{E}[v(x, y')|m]$;*

(iv) *$\mathcal{P}^*$ is derived by α_1^*, α_2^* and β^* consistently with Bayes' rule whenever it is possible. Otherwise, $\mathcal{P}^*$ is any probability distribution satisfying Requirement 1.*

In addition, we assume the following tie-breaking rules; one for the voter, and the other for the medium. Then, we focus on PBEs satisfying the tie-breaking rules in the subsequent analysis.

Requirement 2 ***Tie-breaking Rules***

(i) *Given the voter's posterior belief $\mathcal{P}$, if y_1 and y_2 are indifferent for the voter, then he votes for each candidate with probability $\frac{1}{2}$.*

(ii) *Given a policy pair x such that $x_1 = x_2$. Then, the medium discloses the information.*

In the subsequent analysis, we consider whether the median voter theorem holds as a reference point. We define the median voter theorem in this context as follows.

Definition 2 ***Median Voter Theorem***

(i) *We say that the strict median voter theorem holds if there exists a unique PBE in which $\alpha_1^* = \alpha_2^* = 0$.*

(ii) *We say that the weak median voter theorem holds if there exists a PBE in which $\alpha_1^* = \alpha_2^* = 0$.*

That is, we require the existence of a PBE in which both the non-policy-type candidates propose the voter's ideal policy. To simplify the description, an equilibrium in which the non-policy-type candidates propose x_1 and x_2 for certain is called (x_1, x_2) *equilibrium*.

probability	(θ_1, θ_2)	proposed policy pair	winner	equilibrium policy
p^2	(θ_N, θ_N)	$(0, 0)$	1 or 2	0
$p(1-p)$	(θ_N, θ_P)	$(0, l)$	1	0
$(1-p)p$	(θ_P, θ_N)	$(r, 0)$	2	0
$(1-p)^2$	(θ_P, θ_P)	(r, l)	1	r

Table 1: Equilibrium outcomes in the benchmark model

3 Benchmark Model: No Media

In this section, we analyze the model without the medium as a benchmark model; that is, the voter can directly observe the proposed policies. In the benchmark model, the median voter theorem holds, and thus, we can say that the voter's ideal policy is supported as the equilibrium outcome unless both candidates are of the policy type.

The voter's equilibrium strategy is straightforward. Because the voter can directly observe the proposed policies, the voter can cast the ballot for the candidate whose policy is closer to his ideal point for certain. Then, the argument for the non-policy-type candidates is the same as in the standard Downsian models. That is, because the voter can directly observe policy pair x, proposing the voter's ideal policy, i.e., $\alpha_i = 0$, is the dominant strategy for the strategic candidates. In other words, the $(0, 0)$ equilibrium is the unique equilibrium. The equilibrium outcomes are summarized in Table 1. We can see that the voter's ideal policy can be supported as the equilibrium policy unless both candidates are of the policy type. The following proposition summarizes the results in the benchmark model.

Proposition 1 *Consider the benchmark model. Then,*

(i) $(0, 0)$ *equilibrium is the unique equilibrium, i.e., the strict median voter theorem holds.*

(ii) *The voter's ideal policy is supported as the equilibrium outcome unless both candidates are of the policy type.*

Proof. All proofs are in Appendix A. ■

4 Manipulated News Model

Now, we move back to the model involving the medium. We refer to this as the *manipulated news model.* We show that the equilibrium outcomes are distorted in favor of the medium through the following two channels. The first channel is the distortion in the voter's behavior; the voter's decision making could be incorrect ex post in any equilibrium when the media bias is not small. The second channel is the distortion in the candidates' behaviors; the candidates have an incentive to win the election by influence the medium's behavior through policy settings. That is, the weak median voter theorem could fail. This is the distortion mechanism generated by the medium that can manipulate the information about the proposed policies.

4.1 Information disclosure stage

In this subsection, we analyze a persuasion game between the medium and the voter given the candidates' proposed policies. First, it is worthwhile to make clear the voter's uncertainty at the beginning of the information disclosure stage. The voter faces uncertainty about the proposed policy pair because of the uncertainty about the candidates' types. For example, suppose that $\alpha_1^* = \alpha_2^* = 0$ are the non-policy-type candidates' equilibrium strategies. The voter knows that either one of the pairs, $(0,0), (0,l), (r,0)$ or (r,l) is proposed in the equilibrium, but he cannot specify which policy pair is actually proposed. This is the voter's uncertainty at the beginning of the information disclosure stage.[6] Therefore, the news from the medium is crucial for the voter to choose the correct candidate in the manipulated news model.

Next, we define, and characterize a *full-disclosure equilibrium* as a reference point. Let $Z(\alpha_1^*, \alpha_2^*)$ be the support of the voter's equilibrium prior, i.e., the set of possible policy pairs from the viewpoint of the voter given the equilibrium strategies α_1^* and α_2^*. This is defined by $Z(\alpha_1^*, \alpha_2^*) \equiv \{x' \in X^2 | Pr.(x = x' | \alpha_1^*, \alpha_2^*) > 0\}$. Let $y^v(x)$ be the voter's ex post correct decision making defined by:

$$y^v(x) = \begin{cases} (1,0) & \text{if } |x_1| < |x_2| \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } |x_1| = |x_2| \\ (0,1) & \text{if } |x_1| > |x_2| \end{cases} \tag{4}$$

Then, we define a full-disclosure equilibrium as follows:

Definition 3 ***Full-Disclosure Equilibrium***
A PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *is a full-disclosure equilibrium if* $\gamma^*(\beta^*(x)) = y^v(x), \forall x \in Z(\alpha_1^*, \alpha_2^*)$.

That is, the full-disclosure equilibrium is a PBE where the voter chooses the preferred candidate for certain on the equilibrium path. There are a few remarks about the full-disclosure equilibrium

[6] In other words, as long as we use Nash concepts, players correctly expect the others' strategies in equilibrium. That is, no one faces strategic uncertainty in equilibrium. In the manipulated news model, the policies proposed are the strategies of the candidates and so the voter correctly expects the candidates' strategies in equilibrium. However, because the voter does not know the types of candidates, she faces uncertainty about the proposed policy pair.

to be made. First, the full-disclosure equilibrium only requires that the voter's decision making is correct ex post on the equilibrium path. Hence, we do not care about the correctness of the voter's decision making off the equilibrium path. Second, we also do not care about the medium's behavior. If the medium "directly" discloses the information, i.e., $\beta^*(x) = x$ for all $x \in Z(\alpha_1^*, \alpha_2^*)$, then this is obviously the full-disclosure equilibrium. However, even if the medium withholds the information, then this behavior by the medium could support the full-disclosure equilibrium. For instance, if the medium withholds the information on the equilibrium path only when the medium observes policy pair $x' \in Z(\alpha_1^*, \alpha_2^*)$, then withholding the information itself is a signal about policy pair x'. That is, information about policy pair x' is "indirectly" disclosed.

Now, we begin to characterize the equilibrium strategies of the voter and the medium. The voter's equilibrium behavior is straightforward. If the media sends a message $m = x$, then the voter completely learns the proposed policies. Hence, the voter's decision making is correct. On the other hand, if the media sends a message $m = \phi$, then the voter's uncertainty about the policy pair could remain. The voter's decision making is based on the posterior belief $\mathcal{P}(\cdot|\phi)$. Therefore, the voter's best response to message $m = \phi$ is characterized as follows:

$$\gamma^*(\phi) = \begin{cases} (1,0) & \text{if } \mathbb{E}[|x_1||\phi] < \mathbb{E}[|x_2||\phi] \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } \mathbb{E}[|x_1||\phi] = \mathbb{E}[|x_2||\phi] \\ (0,1) & \text{if } \mathbb{E}[|x_1||\phi] > \mathbb{E}[|x_2||\phi] \end{cases} \tag{5}$$

Given the voter's equilibrium strategy, consider the medium's strategy. Because we define the preferences of the voter and the medium as (2) and (3), given policy pair x, the voter prefers y_1 to y_2 if and only if $|x_1| \leq |x_2|$, and the medium prefers y_1 to y_2 if and only if $|x_1 - b| \leq |x_2 - b|$. Hence, the space $\mathbb{R}^2$ is divided into the following six regions, as shown in Figure 1:

$$A \equiv \{(x_1, x_2) \in \mathbb{R}^2 | [x_2 > x_1 \text{ and } x_2 > -x_1 + 2b] \text{ or } [x_2 < -x_1 \text{ and } x_2 < x_1\} \tag{6}$$

$$B \equiv \{(x_1, x_2) \in \mathbb{R}^2 | [-x_1 + 2b < x_2 < x_1] \text{ or } [x_2 < -x_1 \text{ and } x_2 > x_1]\} \tag{7}$$

$$C \equiv \{(x_1, x_2) \in \mathbb{R}^2 | x_2 > x_1 \text{ and } -x_1 < x_2 < -x_1 + 2b\} \tag{8}$$

$$D \equiv \{(x_1, x_2) \in \mathbb{R}^2 | x_2 < x_1 \text{ and } -x_1 < x_2 < -x_1 + 2b\} \tag{9}$$

$$E \equiv \{(x_1, x_2) \in \mathbb{R}^2 | x_2 = x_1 \text{ or } x_2 = -x_1 \text{ or } x_2 = -x_1 + 2b\} \tag{10}$$

We call regions A, B and E *agreement regions*, and regions C and D *disagreement regions*. If a proposed policy pair lies in the agreement regions, then the voter's and medium's preferences do not strictly conflict. In region A, both the voter and the medium strictly prefer y_1 to y_2, and in region B, they agree with strictly preferring y_2 to y_1. In region E, either the voter or the medium

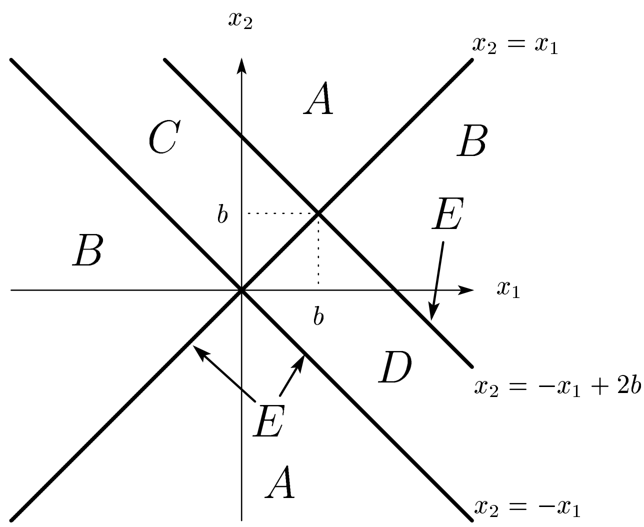

Figure 1: Distribution of preferences

is indifferent between y_1 and y_2. On the other hand, if a proposed pair lies in the disagreement regions, then the voter's and the medium's preferences strictly conflict. In region C, the voter strictly prefers y_1 to y_2, but the medium strictly prefers y_2 to y_1. Similarly, in region D, the voter strictly prefers y_2 to y_1, but the medium strictly prefers y_1 to y_2.

If the medium observes a policy pair in the agreement regions, then disclosing the information is one of the best responses. Conversely, if the medium observes a policy pair in the disagreement regions, then withholding is weakly better than disclosing for the medium. Hence, the medium's equilibrium strategy is characterized as follows:

$$\beta^*(x) = \begin{cases} x & \text{if } x \in (A \cup B \cup E) \cap X^2 \\ \phi & \text{if } x \in (C \cup D) \cap X^2 \end{cases} \tag{11}$$

Hereafter, we focus on equilibria satisfying (11) when we construct equilibria.[7]

The full-disclosure equilibrium is characterized as follows:

Proposition 2 *Consider the manipulated news model with single medium. There exists full-disclosure equilibrium* $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *if and only if either* $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ *or* $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$.

Whether the full-disclosure equilibrium exists depends on whether the voter can correctly infer the medium's motivation behind the withholding. For example, suppose that $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$. That is, at the beginning of the information disclosure stage, the voter can infer

[7] Of course, (11) is not the unique best response for the media. If we show the impossibility results, we do not then restrict the medium's strategy to (11).

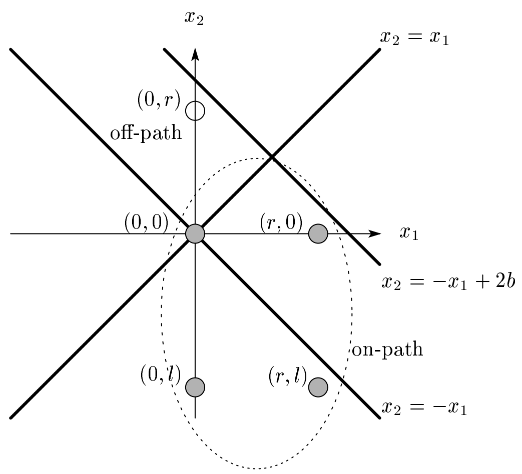

Figure 2: Incorrect decision making for off the equilibrium path policy.

that the medium wants to conceal the information only in the disagreement region C. Given the prior belief of the voter, the withholding itself is a signal showing that the proposed policy pair lies in the disagreement region C. Then, the voter chooses candidate 1 if he observes the withholding. In other words, because the voter can correctly infer the medium's motivation for withholding, the medium does not successfully conceal the unfavorable information on the equilibrium path, and full information disclosure is then possible.

However, if the voter is ambiguous about the medium's motivation behind the withholding, then the medium successfully conceals part of the unfavorable information on the equilibrium path. Suppose that $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$. In the voter's prior belief, there are two explanations for the withholding. The voter cannot distinguish whether the medium tried to conceal the policy pair in disagreement region C or disagreement region D from observing the withholding. Because of this indeterminacy, the voter's decision making is incorrect with positive probability on the equilibrium path. That is, full information disclosure is impossible.[8]

As a corollary of Proposition 2, we obtain the following result.

Corollary 1 *Consider the manipulated news model with single medium. Suppose that $b > \frac{1}{2}r$. Then, in any equilibrium, there exists, at least, one policy pair $x \in X^2$ such that $\gamma^*(\beta^*(x)) \neq y^v(x)$.*

That is, the existence of the medium certainly distorts the voter's decision making for some policy pair. For a non-full-disclosure equilibrium, this claim is obvious. However, this claim is also true for the full-disclosure equilibrium. In any full-disclosure equilibrium with media bias that is not

[8] In the terminology of persuasion games, if $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then there does not exist the *worst case inference* for message $m = \phi$.

small, the voter's decision making at some off the equilibrium path policy pair must be incorrect. In other words, full information disclosure on the equilibrium path is supported by the voter's incorrect decision making off the equilibrium path.

Suppose, for example, that $\alpha_1^* = 0$ and $\alpha_2^* = 0$. By Proposition 2, in this scenario the full-disclosure equilibrium exists because $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ as shown in Figure 2. To support this equilibrium, the voter's response to the withholding must be $\gamma^*(\phi) = y_2$. In this equilibrium, policy pair $x = (0, r)$ is off the equilibrium path, and the voter and the medium disagree with the outcome given that policy pair; the voter prefers candidate 1, but the medium prefers candidate 2. Thus, given the voter's response $\gamma^*(\phi) = y_2$, the medium that observes policy pair $x = (0, r)$ withholds the information, and so candidate 2 wins for certain. That is, the voter's decision making at policy pair $x = (0, r)$ is incorrect ex post.

In summary, the medium successfully conceals part of the unfavorable information when the media bias is not small. Because of this manipulation, the voter's decision making is certainly distorted at some policy pair. In the following subsection, we see how the voter's incorrect decision making affects the behavior of the non-policy-type candidates.

4.2 Policy setting stage

In this subsection, we analyze how the non-policy-type candidates behave in the policy setting stage. The media manipulation generates the following two effects. First, it is less beneficial for the candidates to propose the voter's ideal policy. Second, the candidates have an incentive to win the election by influencing the medium's behavior through policy settings. As a result, even the weak median voter theorem does not hold in the manipulated news model when the media bias is not small.

Depending on the magnitude of the media bias, we consider several cases. Suppose that $0 < b \leq \frac{1}{2}r$. In this case, the media bias is so small that the medium's and the voter's preferences never conflict. Thus, the result is the same as that of the benchmark because all information is disclosed. That is, the strict median voter theorem holds. Then, hereafter, we suppose that $b > \frac{1}{2}r$. In this scenario, the voter and the medium conflict only over policies 0 and r. Hence, policy pairs $(0, r)$ and $(r, 0)$ are in the disagreement regions.

Proposition 3 *Consider the manipulated news model with single medium, and suppose $b > \frac{1}{2}r$. Then the strict median voter theorem does not hold. Moreover, the weak median voter theorem holds if and only if $p \leq \frac{1}{2}$.*[9]

[9] This is the necessary and sufficient condition under the tie-braking rules specified in Requirement 2.

There are the following two contrasts with the benchmark results. First, there exist multiple equilibria. In addition to the $(0, 0)$ equilibrium, there exist $(0, r)$, (r, r) and mixed strategy equilibria.[10] Second, the $(0, 0)$ equilibrium does not always exist; we require the condition that the policy-type candidates are more likely than the non-policy-type candidates.

The multiplicity of equilibrium arises because appealing to the voter becomes less attractive to the candidates because of ex post incorrect decision making by the voter. In the benchmark model, proposing the voter's ideal policy is the dominant strategy for both candidates. Because the voter correctly observes the proposed policy pair, appealing to the voter by proposing $x_i = 0$ is the unique way to maximize the winning probability. Hence, only the $(0, 0)$ equilibrium exists. However, in the manipulated news model, the voter could not correctly recognize the attractiveness of the candidate that proposes the voter's ideal policy because of media manipulation. As a result, the candidate who proposes ex post less attractive policy for the voter could win with positive probability. Therefore, from the viewpoint of the candidates, proposing other than the voter's ideal policy cannot be dominated by proposing the voter's ideal policy.

In this case, there exists (r, r) equilibrium supported by $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. For candidate 2, the winning probability from $x_2 = r$ is $\frac{1}{2}$ in both the benchmark and manipulated news models. Suppose that candidate 2 deviates to proposing $x_2 = 0$. In the benchmark, this deviation strictly improves his winning probability; candidate 2 wins for certain when $x = (r, 0)$. Consequently, the (r, r) equilibrium never exists. However, in the manipulated news model, this deviation does not strictly improve his winning probability. For candidate 2, he is indifferent between proposing $x_2 = r$ and $x_2 = 0$. The winning probability from proposing $x_2 = 0$ is also $\frac{1}{2}$ because the information at $x = (r, 0)$ is withheld by the medium, and then the voter chooses each candidate equally likely. That is, the voter's ex post incorrect decision making at $x = (r, 0)$ because of the manipulation makes this deviation less attractive to candidate 2. Therefore, the (r, r) equilibrium exists. For the same reason, there exists a mixed strategy equilibrium in which the candidates randomize policies $x_i = 0$ and $x_i = r$.

Unlike the benchmark, the $(0, 0)$ equilibrium does not exist when $p > \frac{1}{2}$ in the manipulated news model. This fragility of the $(0, 0)$ equilibrium arises from the incentive of the candidates to win the election by influencing the medium's behavior through policy settings instead of appealing to the voter. For example, suppose that an equilibrium is supported by $\gamma^*(\phi) = y_1$. Given the voter's response, candidate 1 prefers and candidate 2 dislikes media manipulation. Hence, through policy setting, candidate 1 has an incentive to lead the manipulation, which candidate 2 has an

[10] The characterization of each equilibrium is in Appendix B.

incentive to avoid. We refer to these incentives as *influence incentives*. The influence incentives are the main force in breaking down the $(0, 0)$ equilibrium; the $(0, 0)$ equilibrium is collapsed when the influence incentive dominates the incentive to appeal to the voter.

When $b > \frac{1}{2}r$, the $(0, 0)$ equilibrium is supported by $\gamma^*(\phi) = y_2$ when $p \le \frac{1}{2}$. Given the voter's response, candidate 2 has the strong influence incentive to lead the manipulation. Because candidate 2 wins for certain when the proposed policy pair lies in the disagreement regions, candidate 2 wants to induce either policy pair $x = (r, 0)$ or $x = (0, r)$. If $p \le \frac{1}{2}$, proposing $x_2 = 0$ is compatible with the influence incentive of candidate 2 because candidate 1 is more likely to propose $x_1 = r$. However, if $p > \frac{1}{2}$, then candidate 1 is more likely to propose $x_1 = 0$. Hence, proposing $x_2 = 0$ is no longer compatible with the influence incentive of candidate 2; proposing $x_2 = r$ is his best response. That is, the $(0, 0)$ equilibrium is collapsed by the candidate 2's influence incentive when $p > \frac{1}{2}$. Therefore, in this case, the weak median voter theorem does not hold. For the same reason, the candidate 2's influence incentive to avoid the manipulation supports the $(0, r)$ equilibrium when $p \le \frac{1}{2}$.

In summary, the media manipulation distorts the behaviors of the non-policy-type candidates through the discount of the benefit from appealing to the voter and the influence incentives. With the growth of media bias, the incentive of appealing to voters becomes weaker, but the influence incentive becomes stronger. Hence, if the media bias is sufficiently large, the latter dominates the former. Therefore, policy convergence to the voter's ideal policy does not always hold, and several policy pairs can be supported in equilibrium.

4.3 Comparison of equilibrium outcomes

Given the analysis so far, we compare equilibrium outcomes of the manipulated news and the benchmark models. We have already shown that the equilibrium outcomes are distorted when the media bias is sufficiently large. In this subsection, we consider the question of how equilibrium outcomes are distorted because of the media manipulation.

For ease of explanation, we focus on the mixed strategy equilibrium in which $\alpha_1^* = (\frac{1}{2p}, 1 - \frac{1}{2p}, 0)$ and $\alpha_2^* = (\frac{1}{2}, \frac{1}{2}, 0)$ when $b > \frac{1}{2}r$ and $p > \frac{1}{2}$. The equilibrium outcomes are summarized in Table 2. We can observe two kinds of distortions in this equilibrium. The first distortion is from the distortions in the candidates' behavior, and the second one is from the distortions in the voter's behavior. The first is *indirect distortion* and the second is *direct distortion*.

With indirect distortion, the equilibrium outcomes are distorted through distortions in the candidates' behavior. As already mentioned, the non-policy-type candidates have an incentive to propose other than the voter's ideal policy because of the media manipulation. As a result of

probability	proposed policy pair	media	winner	equilibrium policy
$\frac{1}{4}p$	$(0, 0)$	discloses	1 or 2	0
$\frac{1}{4}p$	$(0, r)$	withholds	1 or 2	0 or r
$\frac{1}{2}(1-p)$	$(0, l)$	disclose	1	0
$\frac{1}{4}p$	$(r, 0)$	withholds	1 or 2	0 or r
$\frac{1}{4}p$	(r, r)	discloses	1 or 2	r
$\frac{1}{2}(1-p)$	(r, l)	discloses	1	r

Table 2: Equilibrium outcomes in the mixed strategy equilibrium

the distortions in the candidates' behavior, the policy pairs proposed on the equilibrium path are changed, and so the winning policy is also changed. This is the indirect distortion. In the mixed strategy equilibrium, the indirect distortion appears in the fifth row of Table 2. As shown in Table 1, policy pair $x = (r, r)$ is never proposed on the equilibrium path in the benchmark model. However, because the non-policy-type candidates randomize policies, that policy pair can be proposed on the equilibrium path. Therefore, policy r becomes the winning policy, even if candidate 1 is the non-policy type.

In contrast, the direct distortion is distortion through the voter's behavior. As shown in Proposition 2, the voter's decision making could be incorrect on the equilibrium path. That is, because of the media manipulation, the voter chooses the unfavorable candidate with positive probability. As a result of this incorrect decision making, the winning policy is different from that of the benchmark model. This is the direct distortion. In the mixed strategy equilibrium, we can observe the direct distortion in the fourth row of Table 2. Policy pair $x = (r, 0)$ is proposed on the equilibrium path in both the benchmark and the manipulated news models. As shown in Table 1, policy 0 is the winning policy in the benchmark model. However, the winning policy is r with positive probability in the mixed strategy equilibrium because of the voter's incorrect choice.

We can observe either of the above distortions in all of the equilibria except for the $(0, 0)$ equilibrium. The voter's ex ante expected utility in the manipulated news model is then less than that of the benchmark model, and the winning policy is distorted to the media's ideal policy with positive probability. Therefore, we can conclude that if we measure social welfare by the voter's ex ante expected utility, the presence of the biased medium reduces social welfare.

5 Multiple Media

In this section, we consider a model with multiple mass media. The model is modified as follows. There are two media with media biases $b_1, b_2 \neq 0$. The two media can correctly observe the proposed policy pair x, and each medium j simultaneously sends a message $m_j \in M(x)$ to the voter for $j \in \{1, 2\}$. We assume that the voter can observe both m_1 and m_2 before voting occurs. We say that the media are *like biased* if $b_1 \cdot b_2 > 0$, and *opposing biased* if $b_1 \cdot b_2 < 0$. The results of the multiple media model crucially depend on the directions of the media biases. In the like biased case, equilibrium outcomes are distorted because of the media manipulation similar to the single medium model. However, if the media have opposing-biased preferences, then the information about policy pairs is completely transmitted to the voter; that is the strict median voter theorem holds like the benchmark model.

5.1 Like-biased cases

Without loss of generality, we assume that $0 < b_1 < b_2$.[11] In the like-biased cases, we can obtain the identical results to those obtained in the model with single medium. Because a message from medium 2 never conveys extra information that a message from medium 1 does not disclose, the decision makings of the voter and the candidates do not change from the case where only a message from medium 1 is available. That is, if b_1 is large enough, then equilibrium outcomes are distorted to the direction of the media biases through the direct and indirect distortions.

Proposition 4 *Consider the manipulated news model with like-biased multiple media. Then the equilibrium outcomes are identical to those obtained in the manipulated news model with single medium.*

5.2 Opposing-biased cases

We assume that $b_2 < 0 < b_1$ without loss of generality. In the opposing-biased cases, the results are completely different from those in the model with single medium. If the media are perfectly informed players and have the opposing-biased preferences, then the voter can learn the all information by observing both messages. In other words, if one medium has an incentive to withhold the information, then the other medium definitely has an incentive to disclose it. Suppose, for example, that the proposed policy pair is $x = (r, 0)$. Given the policy pair, medium 1 wants to withholds the

[11]We can obtain the similar results with trivial modification if we assume that $b_1, b_2 < 0$. The details are available from the author upon the request.

information, but medium 2 discloses this information because the voter and the medium 2 share the same preference. Because the voter completely learns the relevant information by observing both m_1 and m_2, the media manipulation observed in the model with single medium never occurs. As a result, the strict median voter theorem holds.

Proposition 5 *Consider the manipulated news model with opposing-biased multiple media. Then, the strict median voter theorem holds.*

This proposition says that monitoring by the media works well if there exist multiple media with opposing-biased preferences. Any deviation that makes the voter worse off is completely reported at least one of the media. Hence, the prediction of the model goes back to that in the benchmark model. This phenomenon is mentioned by Milgrom and Roberts (1986) in the literature of persuasion games.

6 Conclusion

This paper has studied how mass media affect electoral competitions by analyzing a Downsian voting model including mass media that can strategically manipulate the information released to the voter, and specified the distortion mechanism of equilibrium outcomes.

In the manipulated news model with single medium, we have shown that equilibrium outcomes are distorted compared with the model without medium through the distortions in the voter and the candidates behaviors. When the media bias is not small, the medium successfully conceals part of the unfavorable information in any equilibrium. Then, the voter's decision making at some concealed policy pair must be incorrect ex post. Because of the ex post incorrect decision making, appealing to the voter becomes more difficult than the model without a mass medium. We can then observe a variety of policy distributions in equilibrium. Moreover, the non-policy-type candidates also have influence incentives. With the growth of media bias, the influence incentives dominate the incentives of appealing to the voter. That is, when the media bias is large enough, the candidates choose policies in order to influence the medium's behavior, not to appeal to the voter. The weak median voter theorem then fails. This is the distortion mechanism derived by the medium that potentially distorts the information about policy pairs. Even if there exist multiple media, we can observe the identical distortion mechanism when the media biases are like biased. However, if the media have the opposing-biased preferences, then no information distortion occurs. As a result, the strict median voter theorem holds like standard Downsian voting models.

As part of the conclusion of the paper, we now briefly discuss some possible extensions. First, future research should revisit the multiple media model. In this paper, we conclude that the strict median voter theorem holds in the manipulated news model with opposing-biased media. However, this conclusion crucially depends on several assumptions such as (i) the media are perfectly informed players and (ii) the voter can observe two messages without any cost. Therefore, we have to check how this conclusion is robust once one of the above assumptions is relaxed.

The first assumption is relaxed by introducing noise into the media's observation. That is, the medium j can observe policy pair x with probability $q_j > 0$, but cannot observe anything with probability $1 - q_j$ for $j \in \{1.2\}$, and the voter does not know whether each medium observes correct signal like Shin (1994a, b). Obviously, the voter's ex post incorrect decision making is guaranteed in this setup. However, it is not trivial whether the influence incentive dominates the incentive to appealing to the voter. It is an interesting question to characterize the necessary and sufficient condition that the first incentive dominates the latter.

We can relax the second assumption by assuming that the voter chooses one medium and reads only the news from the medium. In this setup, we should incorporate media competitions and discuss how the media biases are endogenously determined in the model. The demand side model by Mullainathan and Shleifer (2005) seems reasonable to describe the media competitions. However, as mentioned in Mullainathan and Shleifer (2005), the results of media competitions depend on the distribution of voters' preferences and purchasing powers. Therefore, we have to justify the model setup if we describe the media competition by their model.

Second, future research should examine different voting model with including media. For example, instead of assuming fully office-motivated, we assume that the strategic candidates are also policy motivated like Wittman (1973). Furthermore, we can relax the full commitment assumption of the winning candidate like Banks (1991) and Harrington (1992). Although these extensions are straightforward in the literature, they seem interesting because it is not trivial how the distortion mechanism specified in this paper changes.

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Appendix A: Proofs of Propositions

Proof of Proposition 1

(i) We show that the following is a PBE:

$$\begin{aligned} \alpha_1^* &= \alpha_2^* = 0 \\ \gamma^* &= \begin{cases} (1,0) & \text{if } |x_1| < |x_2| \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } |x_1| = |x_2| \\ (0,1) & \text{if } |x_1| > |x_2|. \end{cases} \end{aligned} \tag{12}$$

As already mentioned in the body of the paper, it is easily shown that for the non-policy-type candidates, proposing $x_i = 0$ is the dominant strategy; for the non-policy-type candidate 1, it is the weakly dominant strategy, and for the non-policy-type candidate 2, it is the strictly dominant strategy. Therefore, policy pair $x = (0,0)$ is the unique equilibrium policy by the non-policy-type candidates. That is, the median voter theorem holds. (ii) It is obvious from Table 1. ■

Proof of Proposition 2

(Necessity) Suppose, in contrast, that there exists the full-disclosure equilibrium when $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$. Pick $x' \in C \cap Z(\alpha_1^*, \alpha_2^*)$ and $x'' \in D \cap Z(\alpha_1^*, \alpha_2^*)$, arbitrarily. Then, $\gamma^*(\beta^*(x')) = (1,0)$ and $\gamma^*(\beta^*(x'')) = (0,1)$. Because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$. That is, at least, one of the medium that observes policy pair either x' or x'' discloses the information. Without loss of generality, assume that $\beta^*(x') = x'$.

(1) $\beta^*(x'') = x''$**.** In this scenario, $m = \phi$ is an off-the-equilibrium-path message. Let $\gamma^*(\phi) = (q, 1-q)$ be the voter's response to the off-the-equilibrium-path message $m = \phi$, where $q \in [0,1]$. Because the medium that observes policy pair x'' choses $m = x''$ on the equilibrium path, $q = 0$; otherwise the medium has an incentive to deviate from $m = x''$ to $m = \phi$. However, given $\gamma^*(\phi) = (0,1)$, the medium that observes policy pair x' deviates from $m = x'$ to $m = \phi$, a contradiction.

(2) $\beta^*(x'') = \phi$**.** By the hypothesis, $\gamma^*(\phi) = (0, 1)$. However, given the voter's best response, the medium that observes policy pair x' deviates to $m = \phi$, a contradiction.

Therefore, if there exists the full-disclosure equilibrium, then either $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ or $D \cap Z(\alpha_1^*, \alpha_2^*)$.

(Sufficiency) Suppose that either $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ or $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$. Let $S(\mathcal{P}^*(\cdot|\phi))$ be the support of the voter's posterior after observing $m = \phi$. Then, we show the there exists the full-disclosure equilibrium supported by the medium's strategy specified by (11).

(1) $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ **and** $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$**.** Because any point in $Z(\alpha_1^*, \alpha_2^*)$ is in the agreement regions, $\beta^*(x) = x$ for all $x \in Z(\alpha_1^*, \alpha_2^*)$. Therefore, this is the full-disclosure equilibrium because the voter chooses the preferred candidate for certain on the equilibrium path.

(2) $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ **and** $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$**.** Given the medium's equilibrium strategy specified by (11), $S(\mathcal{P}^*(\cdot|\phi)) \subset C$. Hence, $\gamma^*(\beta^*(x)) = (1, 0)$ for any $x \in C \cap Z(\alpha_1^*, \alpha_2^*)$. Therefore, because the medium discloses the information about policy pairs in the agreement regions, this is the full-disclosure equilibrium.

(3) $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ **and** $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$**.** Similar to case (2), $S(\mathcal{P}^*(\cdot|\phi)) \subset D$ given the medium's equilibrium strategy specified by (11), and then $\gamma^*(\beta^*(x)) = (0, 1)$ for all $x \in D \cap Z(\alpha_1^*, \alpha_2^*)$. Therefore, this is the full-disclosure equilibrium. ■

Proof of Corollary 1

Suppose that $b > \frac{1}{2}r$. Then, policy pair $(0, r)$ lies in disagreement region C, and policy pair $(r, 0)$ lies in disagreement region D. By Requirement 2, the voter's best response to $m = \phi$ must be either $\gamma^*(\phi) = (1, 0), (\frac{1}{2}, \frac{1}{2})$ or $(0, 1)$. If $\gamma^*(\phi) = (1, 0)$, then the medium that observes policy pair $(r, 0)$ withholds the information, and then $\gamma^*(\beta^*((r, 0))) \neq y^v((r, 0))$. Similarly, if $\gamma^*(\phi) = (0, 1)$, then $\gamma^*(\beta^*((0, r))) \neq y^v((0, r))$. If $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$, then $\gamma^*(\beta^*(x)) \neq y^v(x)$ for $x = (0, r)$ and $(r, 0)$ because the medium that observes policy pair $(0, r)$ or $(r, 0)$ strictly prefers the withholding. ■

Proof of Proposition 3

It is enough to show that there exist other equilibria other than the $(0, 0)$ equilibrium for proving the failure of the strict median voter theorem. When $p \leq \frac{1}{2}$, there exist $(0, 0), (0, r), (r, r)$ and mixed strategy equilibria. When $p > \frac{1}{2}$, there exist (r, r) and mixed strategy equilibria. The characterizations of such equilibria are in Appendix B. Hence, we show the necessary and sufficient condition for the existence of the $(0,0)$ equilibrium.

(Sufficiency) Suppose that $p \leq \frac{1}{2}$. We show that the following is a PBE. Note that only policy pairs $x = (0, r)$ and $(r, 0)$ lie in the disagreement regions.

$$\begin{aligned}
\alpha_1^* &= \alpha_2^* = 0 \\
\beta^*(x) &= \begin{cases} \phi & \text{if } x = (0,r) \text{ or } (r,0) \\ x & \text{otherwise} \end{cases} \\
\gamma^*(m) &= \begin{cases} (1,0) & \text{if } m = (0,r), (0,l) \text{ or } (r,l) \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } m = (0,0), (r,r) \text{ or } (l,l) \\ (0,1) & \text{if } m = (r,0), (l,0), (l,r) \text{ or } \phi \end{cases} \\
\mathcal{P}^*(x|m) &= \begin{cases} 1 & \text{if } [m = x' \text{ and } x = x' \text{ for any } x' \in X^2] \text{ or } [m = \phi \text{ and } x = (r,0)] \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{13}$$

It is obvious that $\gamma^*(\cdot)$ and $\beta^*(\cdot)$ is the best responses of the voter and the medium, respectively. For candidate 1, the winning probabilities from proposing $x_1 = 0, r$ and l are $1 - \frac{1}{2}p, 1-p$ and $\frac{1}{2}(1-p)$, respectively. Hence, candidate 1 does not deviate from $x_1 = 0$. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $1 - \frac{1}{2}p$, $\frac{1}{2}(1+p)$ and 0, respectively. Because $p \leq \frac{1}{2}$, candidate 2 does not deviate from $x_2 = 0$. Obviously, $\mathcal{P}^*(\cdot)$ is consistent with Bayes' rule on the equilibrium path. Hence, this is a PBE.

(Necessity) Suppose, in contrast, that there exists the $(0,0)$ equilibrium when $p > \frac{1}{2}$. Because $Z(\alpha_1^*, \alpha_2^*) = \{(0,0), (0,l), (r,0), (r,l)\}$, the following two scenarios are possible:

(1) Full disclosure scenario. By Proposition 2, the full-disclosure equilibrium is possible; that is, the voter's decision making is always correct on the equilibrium path. To support the full-disclosure equilibrium, $\gamma^*(\phi) = (0,1)$ is needed; otherwise, $\gamma^*(\beta^*((r,0))) \neq y^v((r,0))$. Hence, the medium sends $m = \phi$ when she observes policy pair $x = (0,r)$; this is off the equilibrium path. Given the voter and the medium's strategies, the winning probabilities of the candidates are same to the equilibrium characterized in the sufficiency part, so if $p > \frac{1}{2}$, then candidate 2 deviates to $\alpha_2 = r$, a contradiction.

(2) Withholding scenario. Suppose that the medium that observes policy pair $x = (r,0)$ is pooling with the medium that observes the policy pair either $x = (r,l)$ or $(0,l)$ by sending $m = \phi$. Because (r,l) and $(0,l)$ are in agreement region A, $\gamma^*(\phi) = (1,0)$ is needed to hold this equilibrium; otherwise, the medium that observes either $x = (r,l)$ or $(0,l)$ deviates. Given the voter and the medium's strategies, candidate 1's winning probability from proposing $x_1 = 0$

is $1 - \frac{1}{2}p$. However, the winning probability from $x_1 = r$ is 1. Hence, candidate 1 deviates to $x_1 = r$, a contradiction.

Therefore, to hold the $(0, 0)$ equilibrium, $p \leq \frac{1}{2}$ is needed. ■

Proof of Proposition 4

Because $0 < b_1 < b_2$, there are the following three cases: (i) $b_2 \leq \frac{1}{2}r$; (ii) $b_1 \leq \frac{1}{2}r < b_2$; and (iii) $b_1 > \frac{1}{2}r$. In cases (i) and (ii), medium 1 discloses all information, and then the voter completely learns the information regardless of the message from medium 2. In case (iii), both m_1 and m_2 withholds the policy pairs in the disagreement regions. Because, in each case, the voter's posterior after observing both m_1 and m_2 is equivalent to that after observing only m_1, the voter's decision making is identical in both scenarios. Because the voter's decision making does not change, the behaviors of the non-policy-type candidates also do not change. As a result, the equilibrium outcomes are identical to those derived in the manipulated news model with single medium. ■

Proof of Proposition 5

First, let us introduce additional notations. Let $\beta_j : X^2 \to M$ be the strategy of medium j for $j \in \{1, 2\}$, and $\gamma : M^2 \to \Delta(Y)$ be the voter's strategy. A PBE is defined as an analogy of the model with single medium.

It is enough to show that for any policy pair $x \in X^2$, the voter's decision making is correct ex post in any equilibrium. If $b \leq \frac{1}{2}r$, then this statement is obvious. Hence, we assume $b > \frac{1}{2}r$ hereafter. Suppose, in contrast, that there exists an equilibrium $(\alpha_1^*, \alpha_2^*, \beta_1^*, \beta_2^*, \gamma^*; \mathcal{P}^*)$ such that there exists a policy pair $x \in X^2$ satisfying $\gamma^*(\beta_1^*(x), \beta_2^*(x)) \neq y^v(x)$. By Requirement 2-(ii), if $x_1 = x_2$, then $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = y^v(x)$. Hence, $x_1 \neq x_2$ is needed. Because $x_1 \neq x_2$, $y^v(x)$ is either $(1, 0)$ or $(0, 1)$. Without loss of generality, assume that $y^v(x) = (1, 0)$. That is:

$$|x_1| < |x_2|. \tag{14}$$

By the hypothesis, $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = (0, 1)$ or $(\frac{1}{2}, \frac{1}{2})$. Suppose that $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = (0, 1)$. To hold $\gamma^*(\beta_1^*(x), \beta_2^*(x)) \neq y^v(x)$, it is necessary that $\beta_1^*(x) = \beta_2^*(x) = \phi$; otherwise, the voter's decision making about x must be correct ex post. Because both media 1 and 2 withhold the information, it implies that:

$$|x_2 - b_1| \leq |x_1 - b_1|; \tag{15}$$

$$|x_2 - b_2| \leq |x_1 - b_2|. \tag{16}$$

However, there exists no policy pair $x \in X^2$ satisfying (14), (15) and (16) simultaneously, which is a contradiction. In the case of $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = (\frac{1}{2}, \frac{1}{2})$, we can derive a similar contradiction. Hence, in any equilibrium, the voter's decision making must be correct ex post for any $x \in X^2$. Therefore, $\alpha_i = 0$ is the unique way to maximize the winning probability for the candidates. As a result, only $(0, 0)$ equilibrium exists. ■

Appendix B: Supplemental Materials (Not for Publication)

B.1 Characterizations of equilibria in the manipulated news model

We construct an equilibrium in which the medium's strategy is specified by (11). To avoid trivial repetitions, we, hereafter, omit descriptions of the medium's equilibrium strategy, the voter's posterior and the best response after observing the disclosure message.

(1) $(0, r)$ **equilibrium.** Suppose that $p \leq \frac{1}{2}$. Then:

$$\begin{aligned} \alpha_1^* &= 0 \\ \alpha_2^* &= r \\ \gamma^*(\phi) &= (1, 0) \\ \mathcal{P}^*(x|\phi) &= \begin{cases} 1 & \text{if } x = (0, r) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{17}$$

For candidate 1, the winning probability from proposing $x_1 = 0$ is 1, so candidate 1 has no incentive to deviate. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $\frac{1}{2}p$, $\frac{1}{2}(1-p)$ and 0, respectively. Thus, this is a PBE as long as $p \leq \frac{1}{2}$. □

(2) (r, r) **equilibrium.** For any $p \in (0, 1)$:

$$\begin{aligned} \alpha_1^* &= \alpha_2^* = r \\ \gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\ \mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (0, r) \text{ or } (r, 0) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{18}$$

For candidate 1, the winning probabilities from proposing $x_1 = 0, r$ and l are $1 - \frac{1}{2}p, 1 - \frac{1}{2}p$ and $\frac{1}{2}(1-p)$, respectively. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $\frac{1}{2}, \frac{1}{2}$ and 0, respectively. Thus, this is a PBE. □

(3) Mixed strategy equilibrium. Suppose that $p \geq \alpha_2^0$. Then:

$$\begin{aligned} \alpha_1^* &= (\frac{\alpha_2^0}{p}, 1 - \frac{\alpha_2^0}{p}, 0) \\ \alpha_2^* &= (\alpha_2^0, 1 - \alpha_2^0, 0) \\ \gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\ \mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (0, r) \text{ or } (r, 0) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{19}$$

For candidate 1, the winning probabilities from proposing $x_1 = 0, r$ and l are $1 - \frac{1}{2}p, 1 - \frac{1}{2}p$ and $\frac{1}{2}(1-p)$, respectively. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $\frac{1}{2}, \frac{1}{2}$ and 0, respectively. Obviously, $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule on the equilibrium path. Thus, this is a PBE. $\square$

B.2 Robustness

In this subsection, we discuss how the results in the model with single medium robust by relaxing the assumptions.

B.2.1 Asymmetry between the candidates: symmetric setup

We have assumed that the candidates are asymmetric in the sense that the preferred policies of the policy-type candidates are different. This asymmetry is an essential assumption to the results; the asymmetry generates the influence incentives. In order to consider the importance of the asymmetry, we modify the model as follows. For $i \in \{1, 2\}$, if candidate i is the policy type, then he proposes $x_i = r$ for certain.[12] That is, the candidates are completely symmetric. Except for this modification, the model setup is identical. The result is as follows.

Proposition 6 *Consider the manipulated news model with single medium and the symmetric candidates. Then, the weak median voter theorem always holds; that is, the* $(0, 0)$ *equilibrium always exists.*

Proof. When $0 < b \leq \frac{1}{2}r$, the medium never withholds the information. Obviously, the weak median voter theorem then holds. Hence, hereafter, we assume that $b > \frac{1}{2}r$. Suppose that $\alpha_1^* = \alpha_2^* = 0$. Then, $Z(\alpha_1^*, \alpha_2^*) = \{(0, 0), (0, r), (r, 0), (r, r)\}$. We also focus on the medium's strategy specified by (11). Because $b > \frac{1}{2}r$, policy pair $x = (r, 0)$ is in disagreement region C and policy pair $x = (0, r)$ is in disagreement region D. Then, $\beta^*((r, 0)) = \beta^*((0, r)) = \phi$. Given the candidates and the medium's strategy, the voter's consistent belief after the withholding is:

$$\mathcal{P}^*(x|\phi) = \begin{cases} \frac{1}{2} & \text{if } x = (r, 0) \text{ or } (0, r) \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Given the consistent posterior, the voter's best response to the withholding is $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. Given the medium and the voter's strategies, it is indifferent for the non-policy-type candidate 1 to

[12] As long as $b > 0$, if candidates 1 and 2 of the policy type choose policy l, then the medium trivially discloses all information.

propose $x_1 = 0$ and $x_1 = r$; both strategies provide the same winning probability $\frac{1}{2}$. The winning probability from proposing $x_1 = l$ is 0. Therefore, candidate 1 has no incentive to deviate from $x_1 = 0$. By the symmetry between the candidates, candidate 2 also never deviates from $x_2 = 0$. Therefore, $(0, 0)$ equilibrium always exists without any restrictions. ■

In contrast with the asymmetric setup, the weak median voter theorem is persistent in the symmetric setup; that is, there exist multiple equilibria, but the $(0, 0)$ equilibrium always exists without any restrictions. As in the asymmetric setup, because the benefit from proposing the voter's ideal policy is discounted, multiple equilibria exist. The persistence of the policy convergence result is the consequence of the symmetric candidates. Because the candidates are symmetric, the candidates do not have the enough influence incentives, which is the main force breaking down the $(0, 0)$ equilibrium. Therefore, we can conclude that the asymmetric setup is essential to have the fragility of the $(0, 0)$ equilibrium. In the next subsection. we discuss what kind of asymmetry is needed to generate the strong influence incentives.

B.2.2 Asymmetry between the candidates: distance vs direction

In the body of the paper we have assumed the following two kinds of asymmetry between the candidates. The first is the *asymmetry in the distance* in the sense that the voter has the strict preference on the policy pair by the policy-type candidates. The second is the *asymmetry in the direction* in the sense that one candidate prefers a positive policy, but the other prefers a negative policy when they are the policy type. In this subsection, we discuss that the asymmetry in the distance is more essential to generate the strong influence incentives.

First, we consider the asymmetry in the distance by the following one-sided setup. The policy space is defined by $X \equiv \{0, r, 2r\}$ with $r > 0$, and assume that if candidate 2 is the policy type, then he always proposes policy $x_2 = 2r$. That is, in this one-sided setup, there is the distance asymmetry, but no direction asymmetry. The one-sided setup replicates the similar results obtained in the body of the paper; the strict median voter theorem holds when $0 < b \leq \frac{1}{2}r$, and the weak median voter theorem could fail when $\frac{1}{2}r < b \leq r$. Moreover, we obtain the stronger result when $b > r$; the $(0, 0)$ equilibrium never exists regardless of the parameters.

Proposition 7 *Consider the manipulated news model with single medium and the one-sided setup. Suppose $b > r$, then the weak median voter theorem never holds.*

Proof. First, we prove the following lemma.

Lemma 1 *Fix an equilibrium* $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *such that* $(C \cup D) \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$*, arbitrarily. Then, for any policy pairs* $x', x'' \in (C \cup D) \cap Z(\alpha_1^*, \alpha_2^*)$*,* $\gamma^*(\beta^*(x')) = \gamma^*(\beta^*(x''))$*.*

Proof of Lemma 1. Suppose, in contrast, that there exists an equilibrium such that for some policy pairs $x', x'' \in (C \cup D) \cap Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$.

(1) $x', x'' \in C$ **or** $x', x'' \in D$**.** Without loss of generality, assume that $x', x'' \in C$. Because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$ must hold. If $\beta^*(x') = x'$ and $\beta^*(x'') = x''$, then $\gamma^*(x') = \gamma^*(x'') = (1, 0)$. Hence, exactly one of either x' or x'' must send $m = \phi$. Without loss of generality, assume that $\beta^*(x') = x'$ and $\beta^*(x'') = \phi$. Then, $\gamma^*(x') = (1, 0)$. In addition, because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\gamma^*(\phi)$ assigns positive probability to choosing y_2. However, given this voter's behavior, the medium that observes policy pair x' has an incentive to deviate to $m = \phi$, a contradiction.

(2) $x' \in C$ **and** $x'' \in D$**.** Again, because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$ must hold. From Proposition 2, the medium's full-disclosure behavior cannot be supported in equilibrium. Then, the medium must withhold the information for exactly one of the policy pair x' or x''. Without loss of generality, assume that $\beta^*(x') = x'$ and $\beta^*(x'') = \phi$. Because $\gamma^*(x') = (1, 0)$, $\gamma^*(\phi)$ assigns positive probability to choosing y_2. However, given this voter's behavior, the medium that observes policy pair x' deviates to $m = \phi$, a contradiction. ■

Proof of Proposition 7. Suppose, by contrast that there exists the $(0, 0)$ equilibrium. That is, $\alpha_1^* = 0$ and $\alpha_2^* = 0$. Then, $Z(\alpha_1^*, \alpha_2^*) = \{(0, 0), (0, 2r), (r, 0), (r, 2r)\}$. By Proposition 2, full disclosure is impossible. Then, consider the following two cases: (i) $r < b \leq \frac{3}{2}r$ and (ii) $b > \frac{3}{2}r$.

(i) Suppose that $r < b \leq \frac{3}{2}r$. That is, policy pairs $(0, r), (0, 2r), (r, 0)$ and $(2r, 0)$ are in the disagreement regions.

(1) $(r, 0)$ and $(r, 2r)$ are separating. Suppose, by contrast, that $\beta^*((0, 2r)) \neq \beta^*((r, 0))$. Because policy pair $x = (r, 0)$ is not pooling with any policy pairs in $Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*((r, 0))) = (0, 1)$. By Lemma 1, $\gamma^*(\beta^*((r, 0))) = \gamma^*(\beta^*((0, 2r))) = (0, 1)$. Then, $\beta^*((0, 2r)) = \beta^*((r, 2r)) = \phi$; otherwise, $\gamma^*(\beta^*((0, 2r))) = (1, 0)$. However, if $\gamma^*(\phi) = (0, 1)$, then the medium that observes policy pair $x = (r, 2r)$ deviates to $m = (r, 2r)$, a contradiction. Therefore, in this sub-case, $\beta^*((0, 2r)) = \beta^*((r, 0)) = \phi$ and $\beta^*((r, 2r)) = (r, 2r)$ must hold. The voter's posterior after observing $m = \phi$ is $\mathcal{P}^*((0, 2r)|\phi) = \mathcal{P}^*((r, 0)|\phi) = \frac{1}{2}$. Because $2\mathcal{P}^*((0, 2r)|\phi) > \mathcal{P}^*((r, 0)|\phi)$, $\gamma^*(\phi) = (1, 0)$. Given the voter and the medium's strategies, for candidate 1, the winning probability from $x_1 = 0$ is $1 - \frac{1}{2}p$. However, the winning probability from $x_1 = r$ is 1, which is a contradiction.

(2) $(r, 0)$ and $(r, 2r)$ are pooling. That is, $\beta^*((r, 0)) = \beta^*((r, 2r)) = \phi$. To hold this equilibrium, $\gamma^*(\phi) = (1, 0)$ must hold; otherwise, the medium that observes policy pair $x = (r, 2r)$ deviates. However, if $\gamma^*(\phi) = (1, 0)$, then candidate 1 deviates to $x_1 = r$, as shown in sub-case (1), a contradiction.

(ii) Suppose that $b > \frac{3}{2}r$. That is, any divergent policy pairs are in the disagreement regions. By Lemma 1, $\gamma^*(\beta^*((0, 2r))) = \gamma^*(\beta^*((r, 2r))) = \gamma^*(\beta^*((r, 0)))$ must hold.

(1) $\beta^*(x) = \phi$ for $x = (0, 2r), (r, 0)$ and $(r, 2r)$. Given the candidates and the medium's strategies, the voter's consistent belief after withholding is:

$$\mathcal{P}^*(x|\phi) = \begin{cases} \frac{p}{1+p} & \text{if } x = (0, 2r) \text{ or } (r, 0) \\ \frac{1-p}{1+p} & \text{if } x = (r, 2r) \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

Given the posterior, the voter's best response to withholding is $\gamma^*(\phi) = (1, 0)$. Then, for candidate 1, his winning probability from proposing $x_1 = 0$ is $1 - \frac{1}{2}p$. However, if candidate 1 proposes $x_1 = r$, then his winning probability is 1. Then, candidate 1 has an incentive to deviate, a contradiction.

(2) $\beta^*((r, 2r)) = \beta^*((r, 0)) = \phi$ and $\beta^*((0, 2r)) = (0, 2r)$. Because $\gamma^*(\beta^*((0, 2r))) = (1, 0)$, by Lemma 1, $\gamma^*(\phi) = (1, 0)$ is needed. However, given $\gamma^*(\phi) = (1, 0)$, candidate 1 has an incentive to deviate to $x_1 = r$ as shown in (1), a contradiction.

(3) $\beta^*((0, 2r)) = \beta^*((r, 0)) = \phi$ and $\beta^*((r, 2r)) = (r, 2r)$. We can derive a contradiction by the same argument in (2).

Therefore, the $(0, 0)$ equilibrium does not exist. ■

Next, we discuss the asymmetry in the direction by considering the following setup. The policy space is defined by $X \equiv \{-r, 0, r\}$, and assume that candidate 2 of the policy type always proposes policy $x_2 = -r$. Except for this modification, the setup is same to that used in the body of the paper. Note that there is the direction asymmetry, but no distance asymmetry. In this symmetric two-sided setup, we can obtain the similar results to those in the body of the paper except that there exists no equilibrium when $b > \frac{1}{2}r$ and $p > \frac{1}{2}$.[13] This nonexistence problem arises from the

[13]The detail is available from the author upon the request.

relatively weak influence incentives because of the symmetric distance of the preferred policies of the candidates.[14] Therefore, we could conclude that the asymmetry in the distance is required to generate the strong influence incentives.

In summary, asymmetry between the candidates, particularly the asymmetry in the distance, is a crucial assumption to holds the fragility of the $(0, 0)$ equilibrium. However, this condition seems weak; it is natural that candidates are asymmetric in their preferred policies.

B.2.3 Tie-breaking rules

The tie-breaking rules specified in Requirement 2 seem to be crucial to the results. While the tie-breaking rule for the voter is well accepted in the literature, for the medium it would seem to be much more controversial. We have assumed that the medium discloses the information whenever the proposed policies are convergent, but there is no strong justification for this behavior. However, if the medium withholds the information even when the proposed policies are convergent, then we face the serious multiple-equilibrium problem; any randomization between policies 0 and r can be an equilibrium strategy.

Although this multiplicity is a serious problem, most of the equilibria are not robust with respect to small perturbations in the medium's behavior. Instead of assuming full disclosure or full withholding, we thus assume that the medium discloses the information about convergent policies with probability $\epsilon \in (0, 1)$. That is, the medium that observes the convergent policy pairs randomizes disclosure and withholding.[15] For easy reference, we call this tie-breaking rule the *ϵ-randomization rule*, and the original the *disclosure rule*. We can show that even if the probability of disclosure ϵ is sufficiently small, then the set of equilibrium policy pairs under the ϵ-randomization rule is equivalent to that under the disclosure rule. Therefore, we can justify focusing on the equilibria satisfying the disclosure rule from the viewpoint of the robustness.

Let us introduce additional notations. Let EP^{ϵ} be the set of equilibrium strategies of the non-policy-type candidates under the ϵ-randomization rule.[16] Note that the disclosure rule is the

[14] This nonexistence problem occurs as long as we focus on equilibria satisfying the tie-breaking rule. Once we relax the tie-breaking rule, then we can find an equilibrium in this parameter range.

[15] Because the result of the election is indifferent for the medium when the proposed policy is convergent, such randomization can be supported as one of the best responses of the medium.

[16] Formally, $EP^{\epsilon} \equiv \{(\alpha_1^*, \alpha_2^*) \in (\Delta(X))^2 |$ there exists$\beta^*, \gamma^*, \mathcal{P}^*$ s.t. $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE, where β^* satisfies the ϵ-randomization rule.$\}$.

1-randomization rule. The set of policy pairs X^2 is partitioned into the following three groups:

$$CP \equiv \{(0,0),(r,r),(l,l)\}; \quad (22)$$

$$AR \equiv \{(0,l),(l,0),(r,l),(l,r)\}; \quad (23)$$

$$DR \equiv \{(0,r),(r,0)\}. \quad (24)$$

That is, CP, AR and DR are the sets of convergent policy pairs, divergent policy pairs in the agreement regions and divergent policy pairs in the disagreement regions when $b > \frac{1}{2}r$, respectively. Let β^ϵ be the generic notation of the medium's strategy satisfying the ϵ-randomization rule. Especially, with abuse of notation, $\beta^\epsilon(x) = (t, 1-t)$ represents that the medium that observes policy pair x discloses the information with probability $t \in [0,1]$, and withholds with probability $1-t$. Let $\mathcal{P}^\epsilon(\cdot|m)$ and $\gamma^\epsilon(m)$ be the voter's posterior belief and the best response given a message m under the ϵ-randomization rule, respectively. Let $\mu_i(x|\alpha_1,\alpha_2)$ be the probability of policy pair x occurs from the viewpoint of candidate i of the non-policy-type under strategies α_1,α_2.

Proposition 8 *Consider the manipulated news model with single medium. Suppose that $b > \frac{1}{2}r$. Then, for any $\epsilon \in (0,1)$, $EP^1 = EP^\epsilon$.*

Proof. Fix $\epsilon \in (0,1)$, arbitrarily. First, show that $EP^1 \subseteq EP^\epsilon$. Take $(\alpha_1,\alpha_2) \in EP^1$, arbitrarily. That is, there exist β^1, γ^1 and $\mathcal{P}^1$ such that $(\alpha_1,\alpha_2,\beta^1,\gamma^1;\mathcal{P}^1)$ is a PBE. There are the following cases: (i) $\gamma^1(\phi) = (1,0)$, (ii) $\gamma^1(\phi) = (0,1)$ and (iii) $\gamma^1(\phi) = (\frac{1}{2},\frac{1}{2})$.

Consider the case where $\gamma^1(\phi) = (1,0)$. Given $\gamma^1(\cdot)$, $\beta^1(\cdot)$ is characterized as follows:[17]

$$\beta^1(x) = \begin{cases} (1,0) & \text{if } x \in AR \cup CP \\ (0,1) & \text{if } x \in DR. \end{cases} \quad (25)$$

Then, define $\beta^\epsilon(\cdot)$ as follows:

$$\beta^\epsilon(x) = \begin{cases} (1,0) & \text{if } x \in AR \\ (\epsilon, 1-\epsilon) & \text{if } x \in CP \\ (0,1) & \text{if } x \in DR. \end{cases} \quad (26)$$

Now, we show that given α_1,α_2 and β^ϵ, $\gamma^\epsilon(m) = \gamma^1(m)$ for any $m \in M$. If $m = x$, then it is

[17] While disclosing and withholding are indifferent for the medium if the proposed policy pair is $x = (0,r)$, without loss of generality, we can focus on this strategy.

obvious that $\gamma^{\epsilon}(x) = \gamma^1(x)$ for any $x \in X^2$. By Requirement 2-(i), because $\gamma^1(\phi) = (1, 0)$:

$$
\begin{aligned}
& \sum_{x \in Z(\alpha_1,\alpha_2)} |x_1| \mathcal{P}^1(x|\phi) < \sum_{x \in Z(\alpha_1,\alpha_2)} |x_2| \mathcal{P}^1(x|\phi) \\
\iff & \sum_{x \in DR} |x_1| Pr.(x|\alpha_1,\alpha_2) < \sum_{x \in DR} |x_2| Pr.(x|\alpha_1,\alpha_2) \\
\iff & \sum_{x \in DR} |x_1| Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon) \sum_{x \in CP} |x_1| Pr.(x|\alpha_1,\alpha_2) \\
& \quad < \sum_{x \in DR} |x_2| Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon) \sum_{x \in CP} |x_2| Pr.(x|\alpha_1,\alpha_2) \\
\iff & \sum_{x \in Z(\alpha_1,\alpha_2)} |x_1| \mathcal{P}^{\epsilon}(x|\phi) < \sum_{x \in Z(\alpha_1,\alpha_2)} |x_2| \mathcal{P}^{\epsilon}(x|\phi)
\end{aligned}
\tag{27}
$$

By Requirement 2-(i), $\gamma^{\epsilon}(\phi) = (1, 0)$. Therefore, given α_1, α_2 and β^{ϵ}, $\gamma^{\epsilon}(m) = \gamma^1(m)$ for any $m \in M$.

Next, show that given $\alpha_j, \beta^{\epsilon}$ and γ^{ϵ}, α_i is the best response of candidate $i \in \{1, 2\}$. The winning probability of candidate 1 given α_2, β^1 and γ^1 is $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) + \frac{1}{2}\sum_{x \in CP} \mu_1(x|\alpha_1,\alpha_2)$, where $AR_1 \equiv \{(0,l),(r,l)\}$. Similarly, the winning probability of candidate 2 given α_1, β^1 and γ^1 is $\sum_{x \in AR_2} \mu_2(x|\alpha_1,\alpha_2) + \frac{1}{2}\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2)$, where $AR_2 \equiv \{(l,0),(l,r)\}$. Note that given α_2, β^1 and γ^1, proposing policy l with positive probability is never an equilibrium strategy. Then, the objective functions of the candidates 1 and 2 are $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) + \frac{1}{2}[1 - \sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2)]$, and $\frac{1}{2}\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2)$, respectively. Because $(\alpha_1,\alpha_2) \in EP^1$:

$$\sum_{x \in AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) \geq \sum_{x \in AR_1 \cup DR} \mu_1(x|\alpha_1',\alpha_2) \text{ for any } \alpha_1' \in \Delta(X) \tag{28}$$

$$\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2) \geq \sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2') \text{ for any } \alpha_2' \in \Delta(X) \tag{29}$$

The winning probability of candidate 1 given $\alpha_2, \beta^{\epsilon}$ and γ^{ϵ} is $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) + (1 - \frac{1}{2}\epsilon)\sum_{x \in CP} \mu_1(x|\alpha_1,\alpha_2)$. Similarly, the winning probability of candidate 2 given $\alpha_1, \beta^{\epsilon}$ and γ^{ϵ} is $\sum_{x \in AR_2} \mu_2(x|\alpha_1,\alpha_2) + \frac{1}{2}\epsilon\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2)$. Because α_1 does not put any positive probability on proposing policy l, candidate 2's winning probability becomes $\frac{1}{2}\epsilon\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2)$. By (29), we can say that α_2 is the best response to $\alpha_1, \beta^{\epsilon}$ and γ^{ϵ}. Furthermore, given $\alpha_2, \beta^{\epsilon}$ and γ^{ϵ}, proposing policy l with positive probability is never an equilibrium strategy of candidate 1. Then, candidate 1's winning probability is $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) + (1 - \frac{1}{2}\epsilon)[1 - \sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2)]$. By (28), α_1 is the best response of candidate 1 given $\alpha_2, \beta^{\epsilon}$ and γ^{ϵ}. Therefore, because $(\alpha_1, \alpha_2, \beta^{\epsilon}, \gamma^{\epsilon}; \mathcal{P}^{\epsilon})$ is a PBE, $(\alpha_1,\alpha_2) \in EP^{\epsilon}$. For the cases of $\gamma^1(\phi) = (0, 1)$ and $(\frac{1}{2}, \frac{1}{2})$, similarly we can show that $(\alpha_1,\alpha_2) \in EP^{\epsilon}$. Thus, $EP^1 \subseteq EP^{\epsilon}$. The converse is also proven similarly; that is, $EP^{\epsilon} \subseteq EP^1$. Therefore, $EP^1 = EP^{\epsilon}$. ■