TIME SERIES ANALYSIS:
NONSTATIONARY AND NONINVERTIBLE DISTRIBUTION THEORY

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PREFACE

This book attempts to describe nonstandard theory for linear time series models that are nonstationary and/or noninvertible. Nonstandard aspects of the departure from stationarity or invertibiltity have attracted much attention in the field of time series econometrics during the last ten years. Since there seem few books concerned with the theory for such nonstandard aspects, I have been at liberty to choose my way. Throughout this book, attention is oriented toward the most interesting theoretical issue; that is, the asymptotic distributional aspect of nonstandard statistics. The subtitle of the book reflects this.

Chapter 1 is a prelude to the main theme. By using simple examples, various asymptotic distributions of nonstandard statistics are derived by a classical approach which I call the *eigenvalue approach*. It turns out that, if more complicated problems are to be dealt with, the eigenvalue approach breaks down, and the introduction of notions such as the Brownian motion, the Ito integral, the functional central limit theorem, and so on is inevitable. These notions are now developed very deeply in probability theory. In this book, however, a knowledge of such probability notions is required only at a moderate level, which I explain in Chapters 2 and 3 in an easily accessible way.

Probability theory, in particular the functional central limit theorem, en-

ables us to establish weak convergence of nonstandard statistics, and to realize that limiting forms can be expressed by functionals of the Brownian motion. However, more important from a statistical point of view is how to compute limiting distributions of those statistics. For this purpose I do not simply resort to simulations, but employ numerical integration. To make a matter possible we first need to derive limiting characteristic functions of nonstandard statistics. To this end two approaches are presented. Chapter 4 discusses one approach which I call the stochastic process approach, while Chapter 5 the other which I call the Fredholm approach. The two approaches originate from quite different mathematical theories, which I explain fully indicating the advantage and disadvantage of each approach for judicious use.

Chapter 6 discusses and illustrates numerical integration for computing distribution functions via inversion of characteristic functions. This chapter is necessitated because a direct application of any computer package for integration cannot do a proper job. Overcoming the difficulty we employ Simpson's rule which can be executed on a desktop computer. Necessity for accurate computation based on numerical integration is recognized, for instance, when comparison has to be made closely of limiting local powers of competing nonstandard tests.

Chapters 7 through 11 deal with statistical and econometric problems to which the nonstandard theory discussed in previous chapters applies. Chap-

ter 7 considers the estimation problems associated with nonstationary autoregressive models, while Chapter 8 with noninvertible moving average models. The corresponding testing problems called the unit root tests are discussed in Chapters 9 and 10, respectively. Chapter 11 is concerned with cointegration, which is a stochastic collinearity relationship among multiple nonstationary time series. Problems discussed in these chapters have an origin in time series econometrics. I describe in detail how to derive and compute limiting nonstandard distributions of various estimators and test statistics.

Chapter 12, the last chapter, is intended to give a complete set of solutions to problems posed at the end of most sections of each chapter. Most of the problems are concerned with corroborating the results described in the text so that one can gain a better understanding of details of discussions.

There are about 90 figures and 50 tables. Most of them are those of limiting distributions of nonstandard statistics. These are all produced by the method described in this book, and include many distributions which have never appeared in the literature. Among those are limiting powers and power envelopes of various nonstandard tests under a sequence of local alternatives.

This book may be used as a textbook for graduate students majoring in econometrics or time series analysis. A general knowledge of mathematical statistics including the theory of stationary processes is presupposed, although the necessary material is offered in the text and problems of this

book. Some knowledge of programming language like FORTRAN and computerized algebra like REDUCE is also useful.

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CONTENTS