Asymmetric Trade Integration, Expectations, and Growth

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Abstract

We consider a many country endogenous growth model with adaptive heterogenous expectations and international trade in complementary capital goods. We analyze the impact of asymmetric trade integration on the world long run steady state. Levels of endogenous variables can differ across countries even if the long run growth rate is common and such differences are solved from the model. We apply heterogenous learning to generate transition dynamics that are also affected by trade policy. The model generates endogenous long run income clubs (country groups in which technology and output levels persistently differ) and endogenous short run growth clubs (country groups that share short run growth experiences).

Key words: Endogenous growth, complementary capital goods, preferential trade agreements, adaptive expectations.

JEL codes: F43, F15.

1 Introduction:

The impact of international trade on economic growth is a topic of empirical and policy importance that has attracted much attention in economic literature. The proliferation of preferential trade agreements (PTAs) over the last fifteen years has motivated new interest in asymmetric trade integration but less attention has been directed toward its dynamic impact.[1] In this paper, we present an endogenous growth model that we then apply to analyze these dynamic effects.

Preferential trade agreements are a classic topic of study in international trade theory and the static trade creation and trade diversion effects of asymmetric integration in competitive trade models are well-known. Both trade creation and trade diversion arise from changes in the location of production. Trade creation refers to a country switching to importing a product from an PTA partner instead of producing the product itself; since the change in the production location in this instance follows comparative advantage trade creation is efficiency and welfare improving. Trade diversion refers to a country switching trade partners and beginning to import from an PTA partner instead of a source outside the PTA. This sort of a change in trade partners is efficiency and welfare reducing, and thus the the overall impact of an PTA depends on the relative sizes of the trade creating and trade diversion effects.

When increasing returns and imperfect competition are taken into account in a static framework additional effects arise. External returns to scale can lead to industry clustering (agglomeration), and internal increasing returns can yield cost reductions if an PTA results in a larger market. The larger market may also increase competition in any imperfectly competitive sectors, thus producing additional gains from an PTA. Further, PTAs may alter the multilateral trade liberalization process by creating competing trade blocs and by changing the bargaining power and positions of individual countries.[2]

Our goal is to formulate an endogenous growth model within which the dynamic impact of asymmetric integration can be studied. We do this by expanding the closed economy growth model of Evans, Honkapohja and Romer (EHR) (1998) and the two country symmetric trade model of Honkapohja and Turunen-Red (HTR) (2002). HTR showed that international trade in capital goods is capable of creating sudden growth jumps (bifurcations) in which countries quickly find their way to a new high growth equilibrium while previous low growth equilibria are eliminated.[3] The impact of active trade policy on

[1] The term asymmetric trade integration here encompasses all PTAs involving trade policy (e.g., free trade areas and customs unions). We do not consider economic unions that include joint decision making regarding monetary policy, factor movements, or other institutional arrangements.

[2] Comprehensive discussions of static effects of PTAs can be found in Baldwin and Venables (1995), Bhagwati, Greenaway and Panagariya (1998), and World Bank (2005). PTAs as "stumbling blocks" for multilateral trade liberalization have been recently discussed in Limao (2006).

[3] Such favorable growth jumps offer one possible cause for the observed correlation of periods of exceptionally high growth and expanding international trade (Hausmann, Pritchett and Rodrik (2005)).

growth, however, was not considered by HTR nor did they allow for structural and policy asymmetries between countries.

The present model expands the HTR analysis by including several countries that may differ from each other in terms of trade policy, size, cost of innovation, and overall productivity of resources. The inclusion of three or more asymmetric countries allows us to study effects of trade integration on several levels. First, direct effects of PTAs can be usefully isolated if the three countries are restricted to be structurally symmetric. We can then ask whether, e.g., a customs union (of two countries) that raises a common trade barrier against the rest of the world inevitably slows down long run growth in the world. Whether such an PTA may benefit the member countries by providing an asymmetric, even if transitory, boost in growth is also of interest.

Second, structural asymmetries between countries allow for the possibility that PTAs may either exacerbate or mitigate the growth impact of other fundamental differences. For example, a country with lower factor productivity entering into an PTA with a more advanced economy may receive a transitory boost in growth while the growth rate of the world economy that may have been adversely affected by the structural asymmetry may also speed up. Notably, empirical evidence suggests that growth effects of PTAs are asymmetric: agreements that take place between advanced industrialized countries ("North-North" PTAs) appear to be growth enhancing for members whereas the growth impact of mixed "North-South" PTAs is inconclusive and, for "South-South" agreements, even negative (Berthelon (2004)).

In its basic structure, our model is analogous to other "idea" growth models. Three production sectors are included (production of aggregate consumption, invention of new capital goods, production of capital goods) and the source of growth is the endogenous invention of new intermediate capital goods. Following EHR, we assume that intermediate capital goods are complementary to each other in final production and, as in HTR, countries are connected through trade in capital good varieties.

Each new innovation contributes to the output of the final consumption commodity and, because of the complementarity of capital goods, improves the marginal productivity of other capital goods. This raises the productivity and value of new capital goods over time and provides increasing incentives for innovation. Balanced growth is nevertheless obtained because the cost for innovation also increases as technology advances. The cumulative investment in innovation and production of capital goods defines the stock of aggregate capital (denoted by Z) in each time period. A production possibility frontier between this aggregate capital and consumption yields an endogenous opportunity cost for aggregate capital that increases as production of capital goods and innovation expand. Relative importance of increasing opportunity cost of aggregate capital and technological complementarity determine whether the world economy exhibits multiple equilibria.

There are several aspects to the growth process that we consider. First, we characterize the steady states of the model. These define the pace of technological advance in the world and thus yield a common rate of long run growth for all countries. This common rate of long run growth is affected by all symmetries and asymmetries between countries, including asymmetries in trade policy (PTAs). Conveniently, the effects of policy and other asymmetries are determined through a growth multiplier that can be separately evaluated. Solving the impact of trade policy on long run growth and establishing the interactions of trade policy with other structural asymmetries are our first tasks. We obtain that, in the symmetric model, an increase in tariffs (whether unilaterally imposed or established by a customs union) slows down long run growth, assuming that there are no other distortions. However, the effect of an expanding customs union on growth is ambiguous because there exist both growth creating and growth diverting effects as new members join an existing PTA. Retaliation by the rest of the world generally lowers long run growth.

Secondly, levels of endogenous variables can differ across nations even if the long run growth rate is common, and any such differences are solved from the model. Asymmetries in trade policy (PTAs) and other exogenous differences between countries are converted into endogenous long run "income clubs"; countries in such clubs exhibit common long run growth but persistently differ in levels of technology and output. A trade bloc that raises its trade barrier against the rest of the world has a negative effect on the technology levels of the outsiders, while retaliation by the nonmembers of an PTA alleviates losses. Asymmetries of trade policy may also exacerbate level differences that are due to other exogenous asymmetries.

Thirdly, we augment the model with transition dynamics that generate short run differences in growth without violating the common long run growth prediction. Typically, growth dynamics are defined in terms of a state variable (physical or human capital, or technology) that evolves along an explicit trajectory under perfect foresight. In these type of models, the initial value of the state variable (initial conditions) determines the growth trajectory, and a steady state is considered stable if it can be approached from a large set of initial values. (An unstable steady state can only be reached from a set of initial conditions that is of measure zero.) In the present model, we do not define traverse dynamics under perfect foresight. Instead, we assume that individuals observe current values of economic variables and form expectations that they adjust as the economy evolves; in this process, errors in expectations are used so as to learn about the future course of the economy. Steady states that are approached through such learning dynamics are called stable with respect to learning. The initial value of the state variable (Z) does not determine the steady state reached by the learning dynamics; instead, any balanced growth path can be, in principle, realized from any initial value of Z.[4,5]

Adjustment of expectations may not be symmetric across countries. Learning heterogeneity is particularly natural when structural heterogeity exists as is the case in our model.[6] For example, when individuals do not possess per-

[4] See Evans and Honjapohja (2001) for a thorough discussion of learning dynamics.

[5] Krugman (1991) and Matsuyama (1991) discuss the distinction between history (initial conditions) and expectations (under perfect foresight) as determinants of the final outcome.

[6] Evidence for heterogeity in expectations is presented by Pfajfar and Santoro (2007).

fect foresight but adjust behavior in response to observations, decisions about investment in innovation and capital goods production will take into account expected returns on such investment. Since the expected return on investment depends on the expected rate of productivity growth, expected productivity growth will have an effect on investment in a country (Helpman (2004, p. 26)). Expectations about future productivity growth are likely to be less optimistic when present factor productivity is lower and costs of innovation are higher than elsewhere. High barriers to international trade may further lower expectations because of reduced access to foreign capital goods and markets. Consequently, realized short run growth in countries that are subject to adverse asymmetries may well fall short of growth experiences elsewhere.

Honkapohja and Mitra (2006) have studied stability conditions for learning dynamics in the presence of structural heterogeneity. In the present context, their results suggest that any asymmetries between countries (including discriminatory PTAs) will affect national transition dynamics, i.e., as suggested above, countries with low levels of technology development may grow more slowly due to the particular asymmetries that afflict them. This suggests that endogenous (steady state) income clubs that involve persistent differences in the level variables across countries and arise from exogenous and policy asymmetries can be associated with short run differences in growth rates (endogenous "growth clubs"). The observed relatively slow growth of some low-income countries (that violates the Solow type convergence hypothesis) may thus at least partly reflect heterogenous learning and associated transition dynamics.

Related Literature: The theoretical and empirial literature that directly addresses dynamic effects of PTAs is rather sparse. The single theoretical treatment of asymmetric trade integration that we have been able to locate is by Walz (1997, 1999). The Walz model is built on the Grossman and Helpman (1991) approach and therefore differs from ours in several respects. First, the level of world technology defines the state variable of the model and the "knowledge-driven" formulation for growth in this world R&D level includes complete international spillovers of knowledge. In contrast, our "lab-equipment" view of innovation does not assume externalities, and new innovations are shared through international trade in capital goods. We do not define transition dynamics using a state variable but assume that individuals must form expectations and learn about growth in a long run equilibrium. Secondly, Walz focusses on a "North-South" world in which only one country (country A) innovates; the two other countries either produce a traditional commodity that does not require intermediate capital goods in its production (country C) or produce both the traditional commodity and an industrial final product (country B) that does require intermediate capital goods. Walz shows that reductions in trade barriers between the more advanced economies A and B can either increase or reduce the growth of technological progress depending on whether the trade liberalization has a trade creating or trade diverting flavor (if A and B raise trade barriers against C, growth declines (trade diversion); if A and B liberalize their mutual trade in intermediate capital goods, growth speeds up (trade creation)).[7]

The significant recent empirical study of the growth effects of PTAs is Berthelon (2004). In contrast to earlier cross-country regression studies (for references, see Berthelon (2004)) that used dummy variables to indicate membership in PTAs and obtained mixed results, Berthelon constructs an explanatory variable that takes into account not only a country's membership in an PTA but also the extent of the new market that is made accessible through the PTA. This allows Berthelon to differentiate between PTAs between small and large countries, and he further distinguishes PTAs according to the degree of development of the members ("North-North", "North-South" and "South-South" agreements). Berthelon obtains that the market size of the partner countries matters and that PTAs contribute positively to growth. North-North type agreements are found to be most potent in improving growth, while the evidence for South agreements suggests that the growth impact may even be negative.[8]

Other literature that does not consider PTAs but discusses related themes includes recent work on international income and growth differences. Waugh (2007) uses a static general equilibrium model to quantify the contribution of trade to relative incomes of countries. His results indicate that asymmetries in trade costs with poorer countries facing higher costs for exports have a significant impact on relative incomes: removing all trade costs would reduce income differences by more than 40% according to Waugh's calculations. While Waugh does not distinguish between distortions of trade patterns that arise from asymmetric trade policy and other costs of trade his results, in combination with those of Berthelon (2004), nevertheless indicate that the role of PTAs is potentially significant. Jones (2007) presents a theoretical model in which complementarity of intermediate inputs plays a central role in explaining large differences in incomes across economies.

A large literature addresses the cross-sectional distribution of income across countries and the observed lack of convergence in incomes and growth between the poorest and richest nations. Sala-i-Martin (1996) discusses the concept of conditional convergence and evidence that supports it. The conditional convergence hypothesis maintains that each country will converge, over time, to a long run steady state that corresponds to a given level of income and rate of growth but that such steady states can differ for economies that are asymmetric in their structural characteristics. Consistently with conditional convergence, one can obtain a world in which some rich countries remain rich for long periods of time, whereas some poor countries remain poor (the "twin peaks" in income (Quah (1996, 1997)). Pritchett (2000, 2006) goes further and argues that the large differences in incomes and growth can be understood in terms of

[7]Rivera-Batiz and Xie (1992) show that a multilateral (GATT type) reduction of a (symmetric) tariff on intermediate capital goods trade is more likely to speed up growth when the number of countries in the world increases. Thus multilateral reductions of trade barriers may have a more positive impact on growth than bilateral or other PTA arrangements.

[8]Kali, Mendez and Reyes (2007) find that the structure of a country's trade pattern affects growth. First, the number of trading partners is observed to be positively correlated with growth and, second, trade concentration among trade partners is positively associated with growth (but more so for poor countries).

growth regimes (convergence clubs) of countries that experience different steady states, each with its own transition dynamics; the longer run growth experience of a particular country then reflects not only the transition dynamics of the country's initial steady state but also the transitions from one growth regime to another. Since the transition dynamics from one state to another can differ, growth processes of countries can widely vary. [9]

The idea of separate growth regimes that represent multiple long run steady states differs from the structure of the present model in that, in our framework, all countries are subject to the same steady state and thus experience a common rate of growth when the long run equilibrium is attained.[10] However, owing to exogenous structural and policy asymmetries, levels of economic variables can differ across countries and these differences persist in the long run equilibrium.[11] We posit that transition dynamics experienced by individual countries can differ, even in the case of a common long run equilibrium, because of heterogeneity in structural parameters and learning dynamics. Further, because learning dynamics involve individuals adjusting expectations and behavior in response to observed realized (temporary) equilibria, country-specific transition dynamics do not necessarily correspond to a linear relationship between a country's initial state variable (or, income) and growth as suggested by the standard (Solow) convergence hypothesis.[12] Our model includes the possibility of multiple long run steady states that are common for all countries. Switches from one such steady state to another can lead to additional divergence in short run growth rates if countries are asymmetric and subject to heterogenous learning.[13]

2 Model:

In this section, we describe our model and characterize the long run equilibrium solutions. While consumer preferences are assumed to be identical and production in all countries is subject to a three sector structure, we allow for several asymmetries across countries. These exogenous asymmetries roughly reflect fac-

[9] Jermanowski (2006) estimates a Markov-switching regression that includes four growth regimes and transition probabilities from regime to regime that depend on the quality of institutions in a country. Empirical evidence for convergence clubs also includes Canova (2004), Desdoigts (1999), and Johnson and Takeyama (2001).

[10] The negative relationship of annual growth rates and per capital income in 1960 documented in Lucas (2007: Figure 2) suggests that open economies (as defined by Sachs and Warner (1995)) may experience convergence toward a common long run steady state.

[11] We call the endogenous groupings of countries that experience different levels of economic variables in the long run "income clubs" so as to distinguish them from the convergence clubs that usually are understood to imply both country-specific differences in levels and growth in the long run.

[12] For evidence of nonlinearity in this relationship, see Kourtellos (2002).

[13] Since we focus on the impact of asymmetries in trade policy, we do not model the foundations of other structural asymmetries between countries. Howitt and Mayer-Foulkes (2004) show that difficult technology transfer and asymmetries in countries' ability to rise to the challenge of a new innovation technology can explain the appearance of convergence clubs. Baldwin, Martin and Ottaviano (2001) construct a model in which geographic agglomeration amplifies the take-off of early industrialized countries leading to persistent income divergence.

tors that have been identified in the literature as contributing toward income and growth divergence.[14]

2.1 Basic Assumptions:

We assume that there are N (≥ 3) countries, indexed by i. The aggregate consumer in each country maximizes the discounted utility expression

$$U_{it} = \sum_{j=0}^{\infty} \frac{\beta^{t+j} C_{i,t+j}^{1-\sigma}}{1-\sigma}, \quad 0 < \sigma < 1, \tag{1}$$

where $C_{i,t+j}$ denotes final consumption in country i in period $(t+j)$. Given a constant interest rate, r, each aggregate consumer's preferred rate of consumption growth, g_c, is obtained from the Euler equation

$$\frac{C_{i,t+1}}{C_{it}} \equiv g_c = [\beta(1+r)]^{1/\sigma}. \tag{2}$$

Financial capital is taken to be freely mobile, so that the interest rate equalizes worldwide.

Final consumption is produced by a competitive production sector according to the production function

$$Y_{it} = \widehat{L}_i^{1-\alpha} \left(\sum_{k=1}^{N} \int_0^{A_{kt}} x_{ikt}(j_k)^{\gamma} \, dj_k \right)^{\phi}, \quad \widehat{L}_i \equiv \psi_i^{\frac{1}{1-\alpha}} L_i . \tag{3}$$

In (3), L_i denotes the (fixed) endowment of immobile labor (country size). The quantity of intermediate capital goods imported from country k is indicated by $x_{ikt}(j_k)$, where j_k indexes varieties of capital goods supplied by producers in country k. The number of different capital goods produced in a country at time t, A_{kt}, defines the technology level in each location at a point in time. Parameter ψ_i represents total factor productivity; differences in ψ_i reflect institutions and policy environments that have an impact on resource allocation in a country and imperfections in international technology transfer (including differing costs for the adoption of new capital goods).[15] Parameter ϕ determines the degree of technological substitutability among capital inputs; if $\phi > 1$, as we assume, all capital varieties are complements in production. The restriction $\alpha = \gamma\phi$ is imposed in order to preserve linear homogeneity of the production process with

[14] According to Quah (1997) and Kali, Mendez and Reyes (2007) trade patterns (who trades with whom) matter for growth. Waugh (2007) emphasizes trade costs in explaining income differences. Cross-country differences in total factor productivity are discussed in Caselli (2005) and Trefler (1993). Alesina and Giavazzi (2006) discuss differences in research productivity between the U.S. and Europe. Effects of country size are analyzed in Alesina, Spolaore and Wacziarg (2005).

[15] Hsieh and Klenow (2007) measure the impact of resource misallocation on productivity. Howitt (2000) explains productivity level differences using technology transfer.

respect to labor and intermediate inputs. The source of growth is the endogenous invention of new capital goods; in (3), output of final consumption grows as each A_{kt} increases over time and, owing to the complementarity of capital goods, the marginal product of each intermediate capital variety improves with growth in each A_{kt}.

We assume that each country (i) may impose trade barriers, denoted by τ_{ik} (≥ 1), against imported capital goods (from country k); for domestic production trade barriers do not exist ($\tau_{ii} = 1$).[16] All tariff revenues are distributed to the consumption sectors as lump sum income.

Final production sectors take domestic prices as given (all prices are measured with respect to the world market price of final consumption). Maximizing profit given technology (3), final goods producers equate the marginal product of each capital good with its domestic rental price.[17] These equations yield the demand for all varieties of capital goods in all countries ($i, k = 1, ..., N$):

$$\tau_{ik} R_{ikt}(j_k) = \widehat{L}_i^{1-\alpha} \left(\sum_{l=1}^{N} \int_0^{A_{lt}} x_{ilt}(j_l)^{\gamma} \, dj_l \right)^{\phi - 1} \alpha x_{ikt}(j_k)^{\gamma - 1}. \tag{4}$$

In (4), $R_{ikt}(j_k)$ denotes, in country i, the rental prices of capital varieties imported from country k, excluding the trade barriers τ_{ik}.

Intermediate capital goods are supplied by separate monopolistic competitors; patent protection is complete so that there is no replicative innovation. A unit of each capital good is produced by converting one unit of aggregate capital (Z) into a specific capital variety. Production is realized at the end of a time period so that, at the end of a period, a capital goods producer in country i receives revenue $R_{kit}(j_i)x_{kit}(j_i)$ from sales in country k. In the beginning of a time period, $x_{kit}(j_i)$ units of Z are needed to produce the capital units that are rented out. The rental cost for the units of general purpose capital over the time period is $r_t p_t^z x_{kit}(j_i)$, where p_t^z is the opportunity cost of Z in final consumption. Each capital goods producer in country i observes the inverse demand obtained from equations (4) and chooses output $x_{kit}(j_i)$ so as to maximize the (end of the time period) profit

$$\pi_{it}(j_i) = \sum_{k=1}^{N} R_{kit}(j_i) x_{kit}(j_i) - r_t p_t^z \left[\sum_{k=1}^{N} x_{kit}(j_i) \right], \quad j_i \in [0, A_{it}]. \tag{5}$$

After substituting (4) into (5), this maximization yields capital producers' mark-up rules for the three markets:

$$R_{kit} = \frac{r_t p_t^z}{\gamma} \equiv R_t(r_t, p_t^z), \quad i, k = 1, ..., N. \tag{6}$$

[16] We interpret τ_{ik} as an ad valorem tariff (usually expressed as $(1 + \tau_{ik})$) but τ_{ik} may include other nonpolicy trade costs as well. In HTR (2002), symmetric tariffs were implicitly represented by a multiplier $c \in [0, 1]$ that gives the fraction of export revenue that accrues to an exporter.

[17] We treat intermediate capital goods as a service flow from durable capital goods. We exclude depreciation from the model.

Since all varieties of capital goods are priced equally, the index j_i for capital goods (produced in country i) is subsequently dropped.

While we assume that the rate of technological progress is the same in all countries in the long run, exogenous asymmetries are reflected in differences of technology levels. We set

$$A_{it} = \theta_i A_{1t}, \quad i = 2, ..., N, \ (\theta_1 \equiv 1), \tag{7}$$

and the proportionality factors θ_i are to be solved from the model.

Equations (4) together with (6) and (7) give the provision of each capital variety in all markets ($i = 1, ..., N, \ k \neq i$):

$$x_{iit} = \widehat{L}_i (A_{1t} S_i)^{\xi} \left(\frac{R_t}{\alpha} \right)^{\frac{1}{\alpha - 1}}, \ \ x_{ikt} = x_{iit} \tau_{ik}^{\frac{1}{\gamma - 1}}, \ \ \xi \equiv \frac{\phi - 1}{1 - \alpha}, \tag{8}$$

where

$$S_i \equiv \sum_k \theta_k \tau_{ik}^{\frac{\gamma}{\gamma - 1}} = \theta_i + \sum_{k \neq i} \theta_k \tau_{ik}^{\frac{\gamma}{\gamma - 1}}. \tag{9}$$

Aggregate output in country i at time t equals

$$Y_{it} = \widehat{L}_i^{1-\alpha} x_{iit}^{\alpha} (A_{1t} S_i)^{\phi} = \widehat{L}_i (A_{1t} S_i)^{1+\xi} \left(\frac{R_t}{\alpha} \right)^{\frac{\alpha}{\alpha - 1}}. \tag{10}$$

According to (8), capital goods imports to a country (x_{ikt}) decrease with trade barriers (τ_{ik}) and increase with country size and total factor productivity ($\widehat{L}_i$) and the level of technology (A_1), *ceteris paribus.*

Terms S_i defined in (9) give an import tariff -deflated sum of the relative technology levels of a country's trade partners, thus reflecting the accessibility of world technology to the aggregate production sector in country i (*openness factor* of country i). In (8), the S_i state the impact of a country's trade policy and trade pattern on domestic capital goods production (x_{iit}) and imports of intermediate capital. Each S_i decreases (thus reducing x_{iit} and x_{ikt}) as tariffs τ_{ik} increase and this effect is the larger the higher the technology level of the trade partners (θ_k) and the larger the contribution of capital goods in aggregate production (γ). Keeping trade policies τ_{ik} fixed, S_i increases if the country itself or its trade partners become more developed (θ increase). If there are no trade distortions ($\tau_{ik} = 1$ for all i, k), then all S_i are equal and indicate the total world technological attainment level in (8) and (10).

The solution for aggregate output in (10) shows that, in addition to institutional and other exogenous factors (ψ_i), a country's total factor productivity depends the country's openness for technology imports (S_i) that magnifies the contribution of the technology level (A_1); when the degree of complementarity among capital goods (ϕ) increases, the contributions of technology and openness to total factor productivity become more important.

The firms that produce intermediate capital goods are treated as the innovators of each original design. For a new capital good variety to be introduced, a fixed innovation cost must be paid. Following EHR (1998) and HTR (2002), we assume that the cost of a new invention, when developed in country i, equals $\upsilon_i^{\varsigma} p^Z j^{\xi}$ $(\varsigma, \xi > 0)$ units of final consumption. In this specification, j^{ξ} equals the cost of an invention in aggregate capital (Z) and $(\upsilon_i^{\varsigma} p^Z)$ converts the cost of each capital unit into consumption. This specification implies that later innovations are more costly but, owing to capital complementarity, they are also more valuable. The parameters υ_i reflect any factors that impact research productivity in a country across sectors, such as differences in initial human capital and variation in institutions and policies that support innovation (including subsidies for innovative activities); ς that is common for all countries determines the dispersion of the innovation costs.[18]

At each time period, the quantity of new innovations is determined by the zero profit condition of the monopolistically competitive capital producers, i.e., in equilibrium, the discounted stream of monopoly rents earned by the inventor of the last capital variety (obtained using (5), (6) and (8)) must equal the fixed cost of this invention:

$$\upsilon_i^{\varsigma} p_t^{z} \theta_i^{\xi} A_{1t}^{\xi} = \sum_{s=0}^{\infty} (1 + r_{t+s})^{-(s+1)} \pi_{i,t+s}, \tag{11}$$

where

$$\pi_{i,t} = (1-\gamma) R_t \left[\sum_{k=1}^{N} x_{ki} \right] = k_i \Omega A_{1t}^{\xi} (r_t p_t^Z)^{\frac{\alpha}{\alpha-1}}, \tag{12}$$

$$k_i(\theta; \tau, \widehat{L}) \equiv \sum_{j=1}^{N} \widehat{L}_j S_j^{\xi} \tau_{ji}^{\frac{1}{\gamma-1}}, \tag{13}$$

and $\Omega \equiv (1-\gamma)\gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}$. The profit (at the end of period t) for each intermediate good produced in country i is given by (12). In addition to the interest rate (r), opportunity cost of general purpose capital (p^Z) and technology level (A_1) that impact producers in all locations identically, asymmetries yield the country-specific multipliers k_i defined in (13). According to (13), profits are positively affected by the productive size of each market $(\widehat{L}_j)$ and the openness and trade pattern of each economy in technology imports (S_j); an increase in the tariff barriers against imports elsewhere (τ_{ji}) reduces k_i. If all trade is free $(\tau_{ji} \equiv 1)$, then all k_i take the value

$$k_i(\theta; \widehat{L}) \equiv \left(\sum_{j=1}^{N} \theta_j \right)^{\xi} \left(\sum_{j=1}^{N} \widehat{L}_j \right), \tag{14}$$

where the first term includes all technology levels and so reflects all other structural asymmetries while the second multiplicative term indicates the total size of the world market that is equally accessible to all.

[18] Alesina and Giavazzi (2006: Ch. 5) discuss differences in research productivity in the U.S. and among European countries. They observe, for example, that "an academic paper costs society almost twice as much in Italy than in the United Kingdom".

The opportunity cost of general purpose capital is determined by the competitive production sectors' trade-off between final consumption and general purpose capital. We assume that both final consumption and aggregate capital are tradable so that a world production possibility frontier can be expressed as

$$Y_t^w = C_t^w + Z_t^w \Gamma\left(\frac{Z_{t+1}^w - Z_t^w}{Z_t^w}\right). \tag{15}$$

In (15), Y_t^w and C_t^w denote the world total output of final consumption and the amount of this output that is directly consumed; Z_t^w equals the world stock of aggregate capital in time period t. The function Γ is a convex cost function that expresses the cost of aggregate capital in consumption units.[19] Equation (15) yields

$$p^Z = -\frac{dC_t^w}{dZ_{t+1}^w} = \Gamma'\left(\frac{Z_{t+1}^w - Z_t^w}{Z_t^w}\right). \tag{16}$$

The stock of capital at time period t is obtained by adding up the cumulative investment in capital goods (K_t^w) and innovation ($Z_{in,t}^w$) so that

$$\begin{aligned} Z_t^w = K_t^w + Z_{in,t}^w &= \sum_{i=1}^{N}\left[\sum_{k=1}^{N}\int_0^{A_{kt}} x_{kit}(j_k)\, dj_k\right] + \int_0^{A_{1t}} j^{\xi} dj + \sum_{i\neq 1}\int_0^{\theta_i A_{1t}} j^{\xi} dj \qquad (17)\\ &= \sum_{i=1}^{N}\theta_i A_{1t}^{1+\xi} k_i \left(\frac{R_t}{\alpha}\right)^{\frac{1}{\alpha-1}} + \frac{A_{1t}^{1+\xi}}{1+\xi}\left(\sum_{i=1}^{N}\theta_i^{1+\xi}\right)\\ &= A_{1t}^{1+\xi}\left[\left(\frac{R_t}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\sum_{i=1}^{N}\theta_i k_i\right) + \frac{1}{1+\xi}\left(\sum_{i=1}^{N}\theta_i^{1+\xi}\right)\right]. \end{aligned}$$

2.2 Balanced Growth:

Steady state solutions are characterized by a constant interest rate (r), a constant opportunity cost of aggregate capital (p^Z) and a common rate of technology growth, defined by

$$g_A \equiv \frac{A_{1,t+1}}{A_{1t}}. \tag{18}$$

Equation (17) implies that the aggregate capital stock grows at the rate

$$g_Z = (g_A)^{1+\xi} \tag{19}$$

at a long run steady state. By (10), aggregate output must grow at the rate g_Z as well and, due to equations (2) and (15), consumption also grows at this same rate (g_Z).

[19] If a unit of foregone consumption converts to a unit of aggregate capital, (15) corresponds to the accumulation equation $Y_t^w = C_t^w + (Z_{t+1}^w - Z_t^w)$. When $\Gamma(.)$ is not an identity function, the opportunity cost of Z^w is not constant, i.e., the production possibility frontier between consumption and general purpose capital is nonlinear.

Using definition (18) in (11) and (12) we obtain the equations ($i = 1, ..., N$)

$$g_A = \left[1 + r - \widehat{k}_i \Omega r^{\frac{\alpha}{\alpha-1}} (p^Z)^{\frac{1}{\alpha-1}}\right]^{\frac{1}{\xi}}, \quad \widehat{k}_i \equiv \frac{k_i}{\upsilon_i^{\varsigma} \theta_i{}^{\xi}}. \tag{20}$$

Since the growth rate g_A is common for all countries at a steady state, the equilibrium values of multipliers $\widehat{k}_i$ in (20) must also be equal for all nations that innovate. Then, at a steady state,

$$\widehat{k}_1(\theta; \tau, \widehat{L}, \upsilon) = \widehat{k}_i(\theta; \tau, \widehat{L}, \upsilon), \quad i = 2, ..., N. \tag{21}$$

According to (21), relative innovation levels θ_i adjust, depending on all policy and other asymmetries, so as to maintain equal profitability of inventions, per unit of final consumption spent in product development, in all countries (recall that $\theta_1 \equiv 1$).[20]

Substituting (19) into (20) and (2) and taking into account equations (16) and (21) we obtain the following equilibrium conditions:

$$g_Z = [\beta(1 + r)]^{1/\sigma}, \tag{22}$$

$$p^Z = \Gamma'(g_Z - 1), \tag{23}$$

$$g_Z = \left[1 + r - \Omega \widehat{k}_1 r^{\frac{\alpha}{\alpha-1}} (p^Z)^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}}, \tag{24}$$

$$\widehat{k}_1(\theta; \tau, \widehat{L}, \upsilon) = \widehat{k}_i(\theta; \tau, \widehat{L}, \upsilon), \quad i = 2, ..., N. \tag{25}$$

These equations determine the steady state solutions for ($g_Z, r, p_Z;\ \theta_2, ..., \theta_N$), given the exogenous parameters ($\tau, \widehat{L}, \upsilon$). The equilibrium rate of technology advance (g_A) is obtained from (19).

While the rate of technology growth is the same in all countries at a long run steady state, levels of technological attainment can vary as determined by the θ_i solutions. Since equations (25) are separable from the rest of the model, equilibrium differences in technology levels can be solved using only information regarding policy and other asymmetries between countries.[21] Given the steady state values of all θ_i, equations (25) yield the *growth multiplier* $\widehat{k}_1$ that appears in (24); this multiplier transmits the impact on growth of any asymmetries in trade policy (τ_i), country size (L_i), total factor productivity (ψ_i), or cost of innovation (υ_i). Via equations (23) and (22) such asymmetries also affect the equilibrium values of p^Z and r. By (11)-(12) and (20), the growth multiplier represents the steady state impact of country-specific heterogeneities on the profits of the monopolistically competitive innovators. It is this effect on the profitability of innovation that is of importance in the equilibrium condition (24).

[20] If complete symmetry is imposed, equations (21) yield $\theta_2 = \theta_3 = 1 (= \theta_1)$.

[21] Separability occurs because capital goods are assumed not to depreciate. If depreciation were included, θ would appear in (23).

By (25), relative technology levels satisfy the equations

$$\theta_i = \left(\frac{k_i(\theta; \tau, \widehat{L})/\upsilon_i^{\varsigma}}{k_1(\theta; \tau, \widehat{L})/\upsilon_1^{\varsigma}} \right)^{\frac{1}{\xi}}, \quad i = 2, ..., N, \tag{26}$$

i.e., technology levels reflect the relative profitability of innovation in each location, taking into account trade policies and equilibrium trade patterns (k_i) and differences in research productivity (υ_i).[22] Differences in research productivity matter more if the dispersion of such productivity differences (ς) increases; the degree of complementarity among capital goods (ϕ) determines the importance of profitability differences since the parameter ξ in (26) is increasing in ϕ.

Differences in technology levels partly determine equilibrium levels of aggregate output since, by (10) and (9),

$$y_i \equiv \frac{Y_i}{\widehat{L}_i}, \quad y_j \equiv \frac{Y_j}{\widehat{L}_j} \;\Rightarrow\; \frac{y_i}{y_j} = \left(\frac{S_i(\theta, \tau)}{S_j(\theta, \tau)} \right)^{1+\xi}. \tag{27}$$

Thus, aggregate output in a country per effective unit of labor is the larger the more open the country's trade policy and the more advanced its trade partners (as indicated by $S_i(.)$). Conversely, the negative impact on y_i becomes the worse the more the country's tariff pattern targets the most technologially advanced countries. The differences become ever more important as complementarity of capital goods (ϕ) increases.

Given (15), total expenditures and revenues (measured in aggregate consumption) equalize worldwide in every time period. Country-specific balances of payments are maintained by allowing consumption levels to adjust to maintain the equality of export revenues and import costs in final consumption and capital goods; due to the balanced growth of output, consumption, and capital in all countries, trade remains balanced at a steady state.

Figure 1 illustrates a possible configuration of world steady states obtained using equations (22)-(25). The upward sloping curve CC graphs the r and g_Z combinations that satisfy (22). Curve TT represents the points (r, g_Z) where the zero profitability condition (24) is maintained, given the opportunity cost of capital obtained from (23) and the technology levels that are determined in (25).[23] The slope of the TT curve depends on the degree of technological complementarity among capital goods (ϕ) and the curvature of the production possibility frontier between consumption and aggregate capital (Γ''). In particular, if $\Gamma'' = 0$, curve TT is upward sloping (complementarity dominates) but when Γ'' is positive the slope of the TT curve can be negative (Appendix 1).

Steady state equilibria are found at the intersections of the CC and TT curves in Figure 1. Depending on the shape and position of the TT curve, multiple equilibria can occur and the rates of long term growth can significantly

[22] Equation (26) is equivalent to $\theta_i/\theta_j = \left[(k_i(\theta; \tau, \widehat{L})/\upsilon_i^{\varsigma})/(k_j(\theta; \tau, \widehat{L})/\upsilon_j^{\varsigma}) \right]^{\frac{1}{\xi}}$.

[23] Thus, the TT curve in Figure 1 applies in the long run analysis (all countries adjust to a long run steady state). Short run dynamics are considered in Section 5.

differ at such equilibria (e.g., Figure 1 features three steady states at $E1$, $E2$ and $E3$). Furthermore, at each steady state, depending on structural heterogeneities between countries, there can be several long run income clubs (country groups that persistently differ in their relative technology and output levels). According to equations (26) and (27), differences in the level of innovation and output between such income clubs are largely influenced by the relative profitability of innovation, the degrees of openness, and the patterns of trade among various countries. The stability properties of all equilibria under adaptive learning are discussed in Section 5.

The level of welfare at a steady state equilibrium depends on the long run growth rate, g_Z, and the initial level of consumption, C_0. Equation (1) implies that, under balanced growth, the lifetime utility of an aggregate consumer equals

$$U = \frac{C_0^{1-\sigma}}{1-\sigma}(1 - \beta g_Z^{1-\sigma}). \tag{28}$$

Thus, higher growth may not translate to higher long term welfare since an increase in g_Z is normally associated with a reduction in C_0. However, in the normal case, a higher rate of long term growth does imply a higher level of lifetime well-being. Accordingly, in Figure 1, the high growth steady state $E3$ is likely to be preferred, in welfare terms, to the lower steady states at $E1$ and $E2$.

3 World Steady States and Trade:

We next analyze equations (22)-(25) under alternative assumptions about trade policy and other structural asymmetries. Determining the the impact of such differences on long run growth and country-specific technology and output levels are our goals.

3.1 The Free Trade Equilibrium:

We begin by characterizing the world free trade equilibrium in which countries may differ in their structural parameters but there are no trade policy induced distortions ($\tau_{ij} = 1$ for all $i, j = 1, ..., N$). This solution illustrates the model and allows us to analyze the effect of national asymmetries on growth and technology levels without considering trade policy.

For simplicity, we assume that n_1 of the N nations (say countries $1, 2, ..., n_1$) are symmetric with parameters $(\widehat{L}_1, \upsilon_1)$, while the remaining n_2 $(\equiv N - n_1)$ countries each possess the parameters $(\widehat{L}_2, \upsilon_2)$. Because of symmetry, we can set $\theta_i = 1$ for countries $1, 2, ..., n_1$ and $\theta_j \equiv \theta$ in the rest of the world. Equations (25) can then be replaced by the equilibrium condition

$$\theta^{\xi} k_1(\theta; \widehat{L}_1) = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{\varsigma} k_j(\theta; \widehat{L}_2), \tag{29}$$

that applies to countries $j = (n_1 + 1), ..., N$. By (13) and (9), terms k_1 and k_j in (29) equal

$$k_1(.) = k_j(.) = n_1 \widehat{L}_1 S_1^{\xi} + n_2 \widehat{L}_j S_j^{\xi}, \tag{30}$$

and

$$S_1(.) = S_j(.) = n_1 + n_2\theta,$$

and thus

$$\theta^F(\widehat{L}, \upsilon) = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{\frac{\varsigma}{\xi}}. \tag{31}$$

By (31), differences in technology levels at a free trade steady state reflect differences in innovation costs (including any subsidies for innovation activities) and and are also influenced by the parameters (ς, ξ) that determine the dispersion of innovation costs and the degree of complementarity between capital goods. Aggregate productivity and the size of markets $(\widehat{L})$ have no impact on the relative technology level, θ^F, because all producers have equal and costless access to all markets. Solution (31) is less than one (countries $(n_1 + 1), ..., N$ produce a smaller fraction of all capital goods) if the innovation cost parameter υ_1 that applies in countries $1, 2, ..., n_1$ is less than υ_2 in countries $(n_1 + 1), ..., N$.[24]

Proposition 1 *i) The relative technology level solution (31) is decreasing in the cost ratio υ_1/υ_2. ii) If $(\upsilon_1/\upsilon_2) < 1$, the θ^F solution is decreasing in ς and increasing in ξ (equivalently, ϕ).*

According to (31), the relative technology level of countries $(n_1 + 1), ..., N$ declines if their relative innovation cost increases (Proposition 1, part i)). If there is asymmetry in innovation costs (e.g., $(\upsilon_1/\upsilon_2) < 1$), the relative position of the country group with the higher costs is the weaker the larger the dispersion of the innovation costs (ς) and the lower the degree of complementarity among capital goods (ϕ) (Proposition 1, part ii)). The latter effect can be understood by considering profitability of innovation: when capital goods are close complements (ϕ high), the value of each innovation is supported by the existence of all other varieties of capital goods and so asymmetries in innovation costs are less damaging to nations in which such costs are relatively high. If, however, productivity of capital goods is less dependent on other varieties (ϕ low), higher innovation costs are bound to have a larger impact on relative technology levels.

The aggregate output ratio for the two country groups is obtained using (27):

$$\left(\frac{Y_1}{Y_j}\right)^F = \frac{\widehat{L}_1}{\widehat{L}_j}, \quad j = n_1 + 1, ..., N. \tag{32}$$

Thus, differences in total factor productivity and the size of each country $(\widehat{L})$ determine the relative output in the country groups. Technology levels have no

[24] The solution is analogous if we allow for additional (symmetric) country groups. In each case, the ratio of technology levels between two country groups is determined by an innovation cost ratio analogous to (31).

impact on the aggregate output ratios because under free trade each country has equal access to all capital goods ($S_1(\theta) = S_j(\theta)$ in (27)).

By (31) and (32), a long run free trade equilibrium features income clubs that reflect asymmetries in innovation costs (determining relative innovation levels) and differences in total factor productivity and market size (determining relative aggregate output between each group).

The rate of growth at a free trade solution is obtained using equations (22)-(24). The growth multiplier $\widehat{k}_1$ in (24) transmits the growth impact of asymmetries in $\widehat{L}$ and υ and, given (30) and (31),

$$\widehat{k}_1 = \left(n_1 \widehat{L}_1 + n_2 \widehat{L}_j\right) \left[\frac{n_1}{\upsilon_1^{\frac{\varsigma}{\xi}}} + \frac{n_2}{\upsilon_2^{\frac{\varsigma}{\xi}}}\right]^{\xi}. \tag{33}$$

We then obtain[25]

Proposition 2 *Assume that the free trade steady state is unique. Then, the free trade rate of growth, g_Z^F, is increasing in the growth multiplier, $\widehat{k}_1$. Therefore, g_Z^F is increasing in all productivity and market size parameters ($\widehat{L}_i,\ i = 1, ..., N$) and decreasing in all innovation costs (υ_i); if $(\upsilon_1/\upsilon_j) < 1$, then g_Z^F is decreasing in innovation cost dispersion (ς) and increasing in complementarity of capital goods (ϕ).*

Proposition 2 yields the expected conclusion about growth when there are no trade distortions: the world economy will grow the faster the larger the productive size of markets and the lower the costs of innovation. When there are asymmetries in innovation costs, complementarity of capital goods reduces the adverse growth impact of high costs whereas larger dispersion of such costs lowers the equilibrium rate of growth. According to (33), changes in innovation costs affect growth the more the larger the country group in which the change occurs.

Figure 1 illustrates Proposition 2. An exogenous change that increases the growth multiplier $\widehat{k}_1$ corresponds to an upward shift in the TT curve (as shown by the dashed curve in Figure 1). If the free trade equilibrium is unique (and such as, e.g., equilibrium $E1$ in Figure 1, ignoring other equilibria in the figure), the new equilibrium solution along the higher (dashed) TT curve involves faster growth. If there are multiple equilibria, Proposition 2 applies locally near each stable equilibrium, i.e., an increase in the growth multiplier $\widehat{k}_1$ shifts each stable equilibrium toward higher growth (in Figure 1, equilibria $E1$ and $E3$ are stable and both move toward higher growth as the TT curve shifts up).[26] If a change in an exogenous parameter is sufficiently large, a bifurcation of equilibria may occur implying a discrete jump in growth. See HTR(2002) for more discussion of such effects.

The following numerical example illustrates the free trade solution.

[25] Proof of Proposition 2 is in Appendix 1.

[26] Stability of equilibria under adaptive learning is discussed in Section 5.

Example 1: Let the model parameters be the following: $\alpha = 0.39$, $\gamma = 0.35$, $\phi = 1.1$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L}_1 = 0.01$, $\xi = 0.16$, $\varsigma = 1.2$ and let $N = 3$ so that $n_1 = 2$ with $\upsilon_1 = 1$ and $n_2 = 1$.

TABLE 1A: $\widehat{L}_2 = 0.01$					TABLE 1B: $\widehat{L}_2 = 0.04$				
υ_2	θ^F	$(Y_1/Y_j)^F$	$\widehat{k}_1$	g_Z^F	υ_2	θ^F	$(Y_1/Y_j)^F$	$\widehat{k}_1$	g_Z^F
1.0	1	1	0.0358	1.11	1.0	1	0.35	0.0715	1.32
1.2	0.64	1	0.0342	1.09	1.2	0.64	0.35	0.0683	1.31
1.5	0.37	1	0.0336	1.08	1.5	0.37	0.35	0.0673	1.30

In agreement with Propositions 1 and 2, technology level θ^F decreases as the innovation cost υ_2 increases and growth speeds up as the market scale $\widehat{L}_2$ becomes larger. The first row of Table 1A establishes a symmetric free trade solution.[27]

3.2 Structural Symmetry and Tariffs:

We next consider a structurally symmetric world in which countries may nevertheless choose divergent trade policies. This allows us to isolate the effects of policy asymmetries (including PTAs) on long run growth and technology and output levels.

We assume that the first n_1 of the N countries form a customs union (union U), while the remaining n_2 $(= N - n_1)$ countries remain nonmembers (forming the rest of the world, R). The members of the customs union impose a common trade barrier against imports of capital goods from other locations ($\tau_{ij} = \tau$ (> 1), $i \in U$, $j \in R$) but maintain free trade within the union ($\tau_{ij} = 1$, $i, j \in U$). We first assume that the nonmembers do not retaliate ($\tau_{ji} = 1$, $i \in U$, $j \in R$) and then add comments about the impact of such retaliation.

3.2.1 Technology and Output Levels:

Given structural symmetry, we set $\widehat{L}_i \equiv \widehat{L}$ and $\upsilon_i \equiv \upsilon$ for all $i = 1, ..., N$, in equations (22)-(25); thus, every country is equally productive and of equal effective size. Then, setting $\theta_i = 1$ for all $i \in U$ and $\theta_j \equiv \theta$ for $j \in R$, equations (25) yield one equilibrium condition

$$k_j(\theta; \tau, \widehat{L}) = \theta^{\xi} k_1(\theta; \tau, \widehat{L}), \tag{34}$$

where $j \in R$ and

$$k_1(\theta; \tau, \widehat{L}) = \widehat{L}(n_1 S_1^{\xi} + n_2 S_j^{\xi}); \quad k_j(\theta; \tau, \widehat{L}) = \widehat{L}(n_1 S_1^{\xi} \tau^{\frac{1}{\gamma-1}} + n_2 S_j^{\xi}), \tag{35}$$

$$S_1 = n_1 + n_2 \theta \tau^{\frac{\gamma}{\gamma-1}}; \quad S_j = n_1 + n_2 \theta. \tag{36}$$

Thus, by (34), the relative technology level between the two country groups (θ) is influenced by the pattern of tariffs against capital goods across countries

[27] The growth solutions in Example 1 are high (no depreciation). Parameter values can be adjusted.

and the overall openness of each market (S_1 and S_j); since there are no other structural asymmetries, innovation costs (υ) and effective size of individual markets ($\widehat{L}$) do not matter. The importance of market openness increases with the complementarity of capital goods (ξ).

Substitution of (35) and (36) into (34) yields[28]

Proposition 3 *Given the external tariff τ (> 1) imposed by customs union U and maintaining free trade in countries R, the long run relative technology level of countries R satisfies the condition*

$$\theta \in \left(\left(\frac{n_1 \tau^{\frac{1}{\gamma-1}} + n_2}{N} \right)^{\frac{1}{\xi}}, 1 \right). \tag{37}$$

Compared to the symmetric free trade solution (where $\theta = 1$), the increase in the relative technology level of the customs union is the larger the higher the tariff, τ.

By restricting imports of capital goods from the rest of the world (without retaliation) a subgroup of countries can raise its share of innovation activities in the world and lower the relative level of innovation elsewhere. This effect comes about because of the changes in the relative prices of imported and domestic capital goods which alter the profitability of innovation in all locations. Equations (35) and (36) give the country-specific multipliers that determine the profitability of new designs in each country in (12) and, of these two expressions, the derivative of multiplier k_j with respect to tariff τ attains a larger negative value than the derivative $\partial k_1/\partial\tau$ at any initial equilibrium at which $\tau > 1$. Thus, while an increase in the tariff τ lowers profitability of innovation also in countries U, the initial reduction elsewhere is larger. In relative terms, therefore, the level of innovation in countries R decreases as tariff τ increases.

Nevertheless, if all countries initially innovate, the customs union cannot eradicate innovation in the rest of the world: by (37), there is a positive lower limit for the level of innovation in countries R, and this lower limit reflects the relative size of these countries' markets in the world (with symmetry of $\widehat{L}$, countries R comprise (n_2/N)th of the total world market of size $N\widehat{L}$). However, the larger the customs union U (the larger n_1 and the smaller n_2), the smaller the lower limit for the relative technology level, θ. The steady state value of θ also decreases as the customs expands.

Proposition 4 *Keeping tariff τ in countries U fixed, the relative technology level of the members of the customs union is an increasing function of the ratio n_1/n_2.*

Proposition 4 is obtained because each additional member to the customs union offers an additional market which, by expanding the reach of free trade

[28] Proof of Proposition 3 is in Appendix 2.

for intermediate capital goods, further stimulates innovation within the union and lowers the steady state value of θ.

The significance of the relative sizes of the two country groups in determining the relative technology level is further emphasized if we allow for an asymmetry such that $\widehat{L}_1 > \widehat{L}_2\ (= \widehat{L}_j,\ \forall j \in R)$. Then, the asymmetry of the $\widehat{L}$ parameters in favor of the customs union further reduces profitability of innovation in countries R and so the θ solution must further decline (see (26)). Modification of the proof of Proposition 3 yields

Proposition 5 *Given $\widehat{L}_1 > \widehat{L}_2$ and $\tau\ (> 1)$, the steady state solution for the relative technology level in the rest of the world satisfies the condition*

$$\theta \in \left(\left(\frac{n_1 \widehat{L}_1 \tau^{\frac{1}{\gamma-1}} + n_2 \widehat{L}_2}{n_1 \widehat{L}_1 + n_2 \widehat{L}_2} \right)^{\frac{1}{\xi}}, 1 \right). \tag{38}$$

Given $\widehat{L}_1 > \widehat{L}_2$, the low limit for θ in (38) is smaller than the corresponding limit in (37). Proposition xx in Section 4 further establishes that the steady state value of θ must decline as $(\widehat{L}_2/\widehat{L}_1)$ decreases. Accordingly, by Propositions 4 and 5, *the larger and more productive the customs union is in relation to the rest of the world, the more it can expect to gain in innovation share.* Conversely, *if a country is small and/or less productive, the innovation advantage gained from a unilaterally imposed tariff ($n_1 = 1,\ n_2 = N - 1$) is correspondingly reduced.*

The degree of complementarity between capital goods also matters. The relative gain in innovation experienced by the customs union tends to be the larger the lower the degree of technical complementarity (ϕ or ξ).[29] This effect occurs because the profitability of innovation in the rest of the world suffers more as a result of the tariff in countries U when the value of each innovation is less supported by the existence of all other varieties of capital goods.

The aggregate output ratio for countries in the customs union and in the rest of the world is obtained using (27): when $\tau > 1$,

$$\frac{Y_1}{Y_j} = \left(\frac{n_1 + n_2 \theta \tau^{\frac{\gamma}{\gamma-1}}}{n_1 + n_2 \theta} \right)^{1+\xi} < 1, \quad j \in R. \tag{39}$$

The reduction in the Y_1/Y_j ratio below its free trade level (= 1) reflects the distortion in the usage of capital goods that follows the introduction of the import tariff in countries U. By (39), the Y_1/Y_j ratio generally decreases when the tariff becomes larger (but the output ratio can go up when τ increases because the subsequent reduction in θ raises the value of Y_1/Y_j). However, the loss in relative output always decreases with the size of the union.[30]

[29] The θ solution to (34) is a decreasing function of ξ if parameters α and γ are kept fixed (Appendix 2). However, because $\alpha = \phi\gamma$, a reduction of ξ with α fixed requires a reduction in γ. Because of this additional effect, the conclusion about the $d\theta/d\xi$ derivative is only suggestive. The low limit for θ in (37) does decrease if ξ decreases. See Example 2 for a numerical example.

[30] The proof of Proposition 7 is in Appendix 3.

Proposition 6 *i) Near free trade, the output ratio (39) is a decreasing function of tariff τ. ii) Keeping the tariff fixed, the output ratio Y_1/Y_j increases as the customs union expands (the ratio n_1/n_2 increases).*

By Proposition 6 part ii), the expansion of innovation that occurs within a larger customs union following the introduction of new members compensates for some of the aggregate output loss that is caused by the tariff distortion in countries U.

The following numerical example illustrates Propositions 3-4 and 6.

Example 2: Assume that $\alpha = 0.39$ and let ξ and γ take the values stated in the tables. The customs union is formed by two countries in Tables 2A and 2B ($n_1 = 2,\ n_2 = 1$); Tables 2C and 2D show the effect of a unilateral tariff ($n_1 = 1,\ n_2 = 2$).

TABLE 2A: $\xi = 0.16, \gamma = 0.35$

τ	θ	Y_1/Y_3
1.1	0.55	0.99
1.2	0.33	0.98
1.4	0.14	0.99

TABLE 2B: $\xi = 0.10, \gamma = 0.37$

τ	θ	Y_1/Y_3
1.1	0.37	0.99
1.2	0.16	0.99
1.4	0.04	0.99

TABLE 2C: $\xi = 0.16, \gamma = 0.35$

τ	θ	Y_1/Y_i
1.1	0.75	0.96
1.2	0.59	0.94
1.4	0.41	0.91

TABLE 2D: $\xi = 0.10, \gamma = 0.37$

τ	θ	Y_1/Y_i
1.1	0.62	0.97
1.2	0.42	0.95
1.4	0.23	0.94

In Tables 2, when compared to the free trade equilibrium, the formation of a customs union reduces both the steady state technology level of the rest of the world (θ) and the aggregate output ratio (Y_1/Y_j). The value of θ decreases as the tariff increases in all tables, but Table 2A shows that the output ratio may also increase as the tariff is raised. Further, the customs union gains more in the relative technology level and loses less in relative output when the union is larger (Tables 2A and 2B versus Tables 2C and 2D). The effect of the tariff on the technology level is larger when capital goods are less compelementary to each other and the reduction in the output ratio is somewhat smaller (Tables 2B and 2D versus Tables 2A and 2C).

In Example 2, the reduction in the innovation level of the rest of the world is larger than the reduction in the custom union's relative aggregate output. This suggests that *by imposing tariffs against imported capital goods a customs union may attain a sizable relative lead in technological innovation at a modest loss of relative production efficiency.* As an empirical application, one may speculate about the past quantitative effects of developed country protectionism against the less developed countries (LDC's). As observed by Waugh (2007), trade costs are significantly skewed against the LDC's, and the pattern of protection is known to cascade (developed country tariffs rise with the stages of production). Such unfavorable trade costs may help explain the present relative disadvantage of LDC's in technological innovation.

3.2.2 Growth:

In addition to changes in the level variables, the establishment of a customs union also alters the world long run growth rate (g_Z). As in Proposition 2, the direction of this change is determined by the growth multiplier $\widehat{k}_1$ in (24). The growth multiplier is a decreasing function of the tariff, which yields[31]

Proposition 7 *Assume that the long run steady state solution is unique. Then, an increase in in the external tariff imposed by the customs union is followed by a reduction in the world rate of growth. The reduction in growth is the larger the higher the union tariff.*

According to Proposition 7, compared to symmetric free trade, the world economy will grow more slowly if a customs union forms, and the negative growth effect is monotonically increasing in the size of the union tariff.

Figure 1 again illustrates. The downward shift of the TT curve from its upper position (the dashed curve) to the position denoted by TT (the solid curve) depicts the negative impact of the union tariff on the profitability of innovation in the long run. The corresponding shift in (stable) equilibria $E1$ and $E3$ toward lower values of g_Z in the figure shows the effect on growth. If there are multiple equilibria, a downward bifurcation may occur. In that case, the reduction in growth could be large.

Example 3 illustrates with some numerical calculations.

Example 3: Assume that $\alpha = 0.39$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L} = 0.01$, $\upsilon_1 = \upsilon_2 = 1$ and $N = 3$. The customs union is formed by two countries in Tables 3A and 3B ($n_1 = 2,\ n_2 = 1$); Tables 3C and 3D show the effect of a unilateral tariff ($n_1 = 1,\ n_2 = 2$).

TABLE 3A: $\xi = 0.16, \gamma = 0.35$

τ	$\widehat{k}_1$	g_Z
1.1	0.0348	1.10
1.2	0.0343	1.09
1.4	0.0338	1.08

TABLE 3B: $\xi = 0.10, \gamma = 0.37$

τ	$\widehat{k}_1$	g_Z
1.1	0.0327	1.06
1.2	0.0324	1.06
1.4	0.0322	1.05

TABLE 3C: $\xi = 0.16, \gamma = 0.35$

τ	$\widehat{k}_1$	g_Z
1.1	0.035	1.10
1.2	0.034	1.09
1.4	0.033	1.07

TABLE 3D: $\xi = 0.10, \gamma = 0.37$

τ	$\widehat{k}_1$	g_Z
1.1	0.033	1.07
1.2	0.032	1.06
1.4	0.031	1.05

[31] The proof of Proposition 7 is in Appendix 2.

All Tables 3 show the increasingly negative effect of a customs union on long run growth as the union becomes more protectionist. The reduction in growth is smaller (albeit from a lower initial level of growth) when capital goods are less complementary to each other (Tables 3B and 3D versus Tables 3A and 3C). This is interesting because Tables 2B and 2D above show that the reduction in the relative technology level of the rest of the world is likely to be more severe (with the customs union correspondingly gaining more in its technology share) when the degree of complementarity is smaller.

The size of the customs union appears to have little effect on growth in Tables 3 (Tables 3A and 3B versus Tables 3C and 3D) although one may expect, in the light of Proposition 7, that the expansion of a customs union should slow down growth. However, in general, the conclusion cannot be clear-cut. This is because the inclusion of additional members in the customs union is followed by two types of growth effects. First, by joining a customs union, a new member attains free access to markets where its exports previously were subject to a tariff. This new market access expands trade (trade creation) and, as a result, innovation becomes more profitable. Therefore, the expansion of the customs union partly speeds up growth (*growth creation*). On the other hand, a trade diversion effect also exists because the new member country must raise tariffs against the rest of the world. This limits access to innovations developed in the rest of the world and so the profitability of innovation within the customs union is reduced; growth must therefore slow down (*growth diversion*). The relative size of the two opposite effects determines whether the expansion of a customs union raises or lowers growth in the long run.

The growth creating effect of an expanding customs union is likely to be the larger the higher the union tariff (because of the larger significance of new market access) *and the lower the level of innovation elsewhere* (because then innovation already largely takes place within the customs union). Thus, because by Proposition 3 an increase in the union tariff lowers the relative technology share of the rest of the world, *a customs union that raises its tariff wall as it expands is more likely to enhance long term growth.* On the other hand, *the negative growth diversion effect is likely to be the larger the more the expanding customs union reallocates innovation toward itself* (the more the value of θ decreases as the union becomes larger) *and the larger the rest of the world* (n_2) *that is subject to the relocation of innovation. The growth diversion effect is likely to be decreasing in the tariff imposed by the customs union* (because when the union tariff is high most of innovation already takes place within the customs union). *Growth diversion may also occur more frequently if the degree of complementarity among capital goods is low* because such complementarity increases the value of each invention and thus tends to stimulate innovation.

An example of the growth diversion effect appears in Tables 3B and 3D when $\tau = 1.1$ (relatively low union tariff) and $\phi = 1.06$ (low degree of complementarity). As the customs union expands (from $n_1 = 1$ in Table 3D to $n_1 = 2$ in Table 3B), growth rate (g_Z) decreases from 1.07 to 1.06. Tables 2B and 2D indicate that the corresponding reduction in the relative technology level of the rest of the world is rather large (from 0.62 in Table 2D when $n_1 = 1$ to 0.37 in Table

2B when $n_1 = 2$); furthermore, since $n_2 = 2$ in Table 2D, the rest of the world that is subject to the reallocation of innovation is also relatively large ($N = 3$). Tables 4 give an example in which the growth creation effect dominates; here, the customs union first comprises five (Table 4B) and then eight countries (Table 4A) out of nine altogether. The largest increase in growth (from 1.47 in Table 4B to 1.49 in Table 4A) occurs when the tariff is relatively high ($\tau = 1.4$) and the technology share of the rest of the world is correspondingly low.

Example 4: Assume that $\alpha = 0.39$, $\xi = 0.16, \gamma = 0.35$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L} = 0.01,\ \upsilon_1 = \upsilon_2 = 1$ and $N = 9$.

TABLE 4A: $n_1 = 8,\ n_2 = 1$

τ	θ	$\widehat{k}_1$	g_Z
1.1	0.45	0.127	1.49
1.2	0.22	0.126	1.49
1.4	0.06	0.125	1.49

TABLE 4B: $n_1 = 5,\ n_2 = 4$

τ	θ	$\widehat{k}_1$	g_Z
1.1	0.61	0.124	1.49
1.2	0.40	0.121	1.48
1.4	0.20	0.118	1.47

By (28), the level of long term welfare at a steady state normally increases if growth is faster. Thus, undistorted trade maximizes well-being in the long run, tariff escalation by a customs union lowers welfare, and changes in the size of a customs union may either lower or raise long run welfare depending on the corresponding effects of growth. However, as shown in Section 5 below, short run rates of growth may differ across countries even as the long run steady state features a common rate of growth. For example, since the relative technology share of countries in a customs union increases as the customs union forms, the customs union is likely to experience an asymmetric boost in growth at least in the short run; depending on the time horizon of the customs union, such an acceleration in growth may motivate the formation of the customs union in the first place. Further, since a customs union can cause a significant shift in innovation from the rest of the world into the union countries (especially if capital goods are not particularly complementary to each other), there may be additional country-specific gains from innovation activities that have not been considered here (e.g., learning from doing, improvement in total factor productivity) and the potential for such gains may further motivate trade distortions. Along these lines, Propositions 3-7 suggest that imports of capital goods are more likely to be restricted in countries that are large and highly productive; such restrictions are also likely to be directed against smaller and less productive nations.

The above Proposition 7 (a higher tariff in a customs union lowers growth) and the contrasting ambiguity of the growth and welfare consequences of changes in the size of customs unions are consistent with the mixed empirical evidence of Berthelon (2004) and others regarding the growth implications of PTAs. While we can expect that the formation of a customs union reduces long term growth when compared to the free trade outcome (Proposition 7), there is little reason to believe that the growth consequences of an additional country's joining an existing PTA should be uniformly positive or negative for any country in the short run or the world in the long run. The overall impact depends on the relative magnitude of growth creating and growth diverting effects and these, in turn, depend on the size of the markets involved, the tariff barrier imposed by the PTA, and the reallocation of innovation that follows the adjustment in the PTA membership.

Notably, Berthelon (2004) found strong evidence supporting the importance of new market access as fostering growth (growth creation). Berthelon's observation that "North-North" PTAs are more likely to speed up growth and "South-South" PTAs may even lower growth are also roughly consistent with the above discussion (growth creation is more likely when highly innovating countries remove mutual barriers and the relative level of innovation in the rest of the world is low; a customs union of countries in which the level of innovation is low is more likely to reduce growth since access to capital goods from the mroe advanced rest of the world is further limited).

3.2.3 Retaliation:

Countries outside the customs union may retaliate by raising a tariff wall of their own. Suppose, as above, that countries $1, 2, ..., n_1$ form customs union U with an external tariff equal to τ_U (> 1) and let countries $(n_1 + 1), ..., N$ retaliate by forming a competing customs union R which imposes the external tariff τ_R (> 1).

The presense of the retaliatory tariff does not alter the content of Propositions 4 and 6-7 above, i.e., a higher union tariff, τ_U, implies slower growth in the long run, the relative technology level of countries in R and the output ratio Y_j/Y_1 decrease if union U expands, and the output ratio Y_1/Y_j is a decreasing function of tariff τ_U near free trade. As for Proposition 3, the relative technology level of countries R remains a decreasing function of τ_U even when retaliation occurs ($\tau_R > 1$) but condition (37) must be replaced by the weaker restriction

$$\theta \in \left(\tau_U^{\frac{1}{(\gamma-1)\xi}}, \tau_R^{\frac{1}{(1-\gamma)\xi}} \right) \tag{40}$$

that does not take into account the relative size of the markets in the two country groups (Appendix 3). However, condition (40) is still useful in that provides an upper limit for the technology ratio, θ, that is larger than one. I.e., tariff retaliation by countries R may raise the technology ratio *above* its free trade value ($\theta = 1$) even when $\tau_U > 1$.

The following proposition demonstrates that retaliation always improves the relative technology position of countries R and that the aggregate output ratio Y_j/Y_1 and the long run growth rate will decline at least when the tariffs are not too high.[32]

Proposition 8 *Let customs union U impose an external tariff τ_U (≥ 1), and let τ_R (≥ 1) be the external tariff in countries R. i) Then, at any τ_U, the long*

[32] The proof of Proposition 8 is in Appendix 3.

run technology level of countries R is an increasing function of the retaliation, τ_R. ii) Near free trade, the output ratio Y_1/Y_j, $j \in R$, increases in τ_R but, given $\xi < 1$ (equivalently, $\phi < 2 - \alpha$), the rate of long run growth decreases as τ_R becomes larger.

That the competing customs union R gains a larger relative technology share by retaliating is not surprising since the retaliatory tariff raises profitability of innovation in countries R and therefore attracts a larger proportion of future innovation there. Assuming that the tariffs remain low, Proposition 8 shows that the aggregate output ratio and the rate of steady state growth respond as may be expected. The condition on the complementarity parameter ($\phi < 2-\alpha$) allows for a very high degree of complementarity between capital goods and is only a sufficient (but not necessary) condition.

Example 5 offers some numerical experiments.

Example 5: Assume that $\alpha = 0.39$, $\xi = 0.16, \gamma = 0.35$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L}_1 = \widehat{L}_2 = 0.01$, $\upsilon_1 = \upsilon_2 = 1$ and $N = 3$. Customs union U comprises two countries in Tables 5A and 5B ($n_1 = 2,\ n_2 = 1$); in Tables 5C and 5D, customs union U includes only one country ($n_1 = 1,\ n_2 = 2$).

TABLE 5A: $\tau_U \equiv 1.2$

τ_R	θ	Y_1/Y_3	$\widehat{k}_1$	g_Z
1.0	0.33	0.98	0.0343	1.09
1.1	0.44	1.03	0.0329	1.07
1.2	0.56	1.07	0.0318	1.06
1.4	0.80	1.12	0.0304	1.04

TABLE 5B: $\tau_U \equiv 1.4$

τ_R	θ	Y_1/Y_3	$\widehat{k}_1$	g_Z
1.0	0.14	0.99	0.0338	1.08
1.1	0.19	1.04	0.0323	1.07
1.2	0.24	1.08	0.0312	1.05
1.4	0.34	1.16	0.0295	1.03

TABLE 5C: $\tau_U \equiv 1.2$

τ_R	θ	Y_1/Y_3	$\widehat{k}_1$	g_Z
1.0	0.59	0.94	0.0339	1.08
1.1	1.07	0.94	0.0326	1.07
1.2	1.80	0.94	0.0318	1.06

TABLE 5D: $\tau_U \equiv 1.4$

τ_R	θ	Y_1/Y_3	$\widehat{k}_1$	g_Z
1.0	0.41	0.91	0.0329	1.07
1.1	0.74	0.91	0.0313	1.05
1.2	1.25	0.89	0.0304	1.04

Example 5 expands the experiments of Tables 2 and 3 in which only one tariff was present: in Tables 5, the countries that do not belong to customs union U retaliate. In Table 5A, countries U choose the tariff $\tau_U \equiv 1.2$ and in Table 5B the tariff is higher ($\tau_U \equiv 1.4$). For completeness, the first row of both tables states the long run steady state values of θ, Y_1/Y_3, and g_Z, without retaliation (this row is obtained from Tables 2 and 3).

When the retaliatory tariff (τ_R) increases, the equilibrium level of innovation outside union U increases in both Tables 5A and 5B (Proposition 8i)). Furthermore, illustrating a wider scope for Proposition 8 part ii), the output ratio Y_1/Y_3 increases with τ_R and the long run growth rate decreases for all tariffs considered in Tables 5A and 5B (not only near free trade). In this numerical example, therefore, the distortionary impact of the retaliatory tariff that works toward raising the output ratio and lowering the growth rate remains stronger than the opposite effect that appears when the allocation of innovation adjusts (θ increases when τ_R increases) for all tariffs considered and not only near free trade. Table 5B shows that a higher union tariff can have a considerable impact on the allocation of innovation (the relative technology ratio θ is much lower in Table 5B than in Table 5A) but the effects of the higher tariff on the output ratio and long term growth appear much smaller.

Tables 5C and 5D offer a comparison in which the customs union U is smaller (unilateral tariff). According to Propositions 4 and 6, the equilibrium solutions for the technology level should be higher than in the corresponding Tables 5A and 5B and the output ratio Y_1/Y_3 in Tables 5C and 5D should be lower, and these effects are confirmed. Tables 5B and 5D also demonstrate the growth creating effect of a larger customs union: for each τ_R, the value of g_Z is larger in Table 5B (union U inclues two thirds of the world market) than in Table 5A (union U contains one third of all markets). As predicted earlier, this effect is stronger in Tables 5B and 5D where the union tariff (τ_U) is larger; in Tables 5A and 5C, growth creation dominates only when $\tau_R = 1$.

Tables 5 further emphasize the important role that the size of a customs union and its markets play in determining the impact of an PTA. Since union U is smaller in Tables 5C and 5D than in Tables 5A and 5B, its tariff wall has much less effect on innovation in the rest of the world (the initial decline in θ in Tables 5C and 5D is smaller than in Tables 5A and 5B), and even a modest retaliation results in a significant correction in the equilibrum value of θ. A small customs union is therefore much less likely to significantly gain in new innovation and, according to Tables 5C and 5D, the cost in reduced relative output for the union remains even as the rest of the world retaliates ($Y_1/Y_j < 1$ in Tables 5C and 5D whereas Y_1/Y_j significantly increases with retaliation in Tables 5A and 5B).

Overall, Propositions 3-8 indicate that, compared to a symmetric free trade equilibrium in which all (symmetric) countries innovate at the same rate and produce an equal amount of aggregate output, *the establishment of a customs union will lead, in the long run, to the appearance of income clubs whose membership is defined by trade policy:* members of the customs union will produce a relatively larger share of innovations in the world (with the effect being the larger the larger the customs union, the more productive its members, the larger the union markets and the less complementarity there is among capital goods) while supplying a relatively smaller share of the aggregate output (with this reduction in output being the smaller the larger the customs union). Countries that are not members of the customs union will see a reduction in innovation activity with the reduction being potentially large if the markets of these countries are small and total factor productivity low. While retaliation can increase the relative technology share of the nonmembers, it is likely to lower the rate of long run growth which affects all countries (not only the members of the customs union).

Reversing our viewpoint, we may reinterpret our results so as to consider the impact of *unilateral and multilateral reductions of tariffs.* By Propositions 7 and

8, unilateral tariffs and tariffs imposed by customs unions slow down growth. Conversely, therefore, *reductions of unilateral or common tariffs in a customs union will speed up growth*, assuming that no other distortions are present, and this is observed in Examples 3 and 5.[33] In Tables 3 (with no retaliation), there is little difference in the improvement of the long term growth rate as the union tariff is reduced so that the number of countries participating in the tariff reduction does not appear to be significant. In Tables 5 in which a customs union faces retaliation by the rest of the world the increase in the steady state rate of growth is larger when both the union and retaliatory tariffs decline, i.e., here, a *multilateral reduction of trade restrictions yields a larger boost in growth than a partial reduction.* This conclusion is in the spirit of Rivera-Batiz and Xie (1992) who argued, using a knowledge-driven model of innovation, that an increase in the number of countries participating in a tariff reduction increases the likelihood of the effect on growth being positive.

4 Asymmetry and Tariffs:

Under free trade, country-specific asymmetries in technology costs, total factor productivity, and the size of markets determine relative technology and aggregate output levels (long run income clubs) and also impact the common rate of long run growth. When structural asymmetries are excluded, differences in tariff policy and the size of the country groups following a given policy appear as the important explanatory factors. Interactions of structural and policy generated asymmetries are considered in this section.

5 Heterogenous Learning Dynamics:

In this section, we formulate adaptive learning dynamics to augment the description of the long run steady states in (22)-(25). We ask what the growth dynamics would be if individuals form expectations about growth, make economics choices based on these expectations, and then learn about the equilibrium rate of growth based on the observed short run behavior. As expectations are adjusted based on observations, a dynamic process is obtained through which some long run equilibria are distinguished as being stable under adaptive learning. Below, we first formulate the adaptive learning dynamics and consider the stability of equilibria. We then discuss the characteristics of the short run dynamics.[34,35]

[33] Rivera-Batiz and Romer (1991) and Rivera-Batiz and Xie (1992) have shown that, in a knowledge driven model of innovation, the long run rate of growth is not necessarily a monotonic function of a tariff distortion.

[34] This discussion applies the structurally symmetric model of Section 3.2.

[35] For more detailed discussions of adaptive learning dynamics, see Evans and Honkapohja (2001) and Honkapohja and Mitra (2006).

5.1 Stability under Heterogenous Learning:

In our present context, there are two representative learners: a country that is a member of customs union U (country 1, say) and a country that remains outside the union (country 2, say). Because of the symmetry among countries in union U and in the rest of the world, respectively, we define the learning dynamics by considering countries 1 and 2 only (individuals within each country are assumed to hold the same expectations).

A model of learning dynamics consists of two components: i) a mapping from expectations to a temporary equilibrium, and ii) a learning rule that describes the updating of expectations based on the observed past. The mapping from expectations to a temporary equilibrium is constructed as follows.

At time period t, producers hold expectations about growth in the world economy; we denote these expectations by g_{1t}^e for countries U and g_{2t}^e for the rest of the world. Based on expectations, firms project profitability of investment and make plans to invest in innovation and production of capital goods. In part, the investment plans reflect expected productivity of aggregate production as this productivity affects returns to investments and depends on future availability of capital goods. Investment plans of producers interact in financial markets where the demand for aggregate capital determines a temporary equilibrium interest rate; this interest rate is consistent with expected zero profitability for innovation in all locations. In addition to the interest rate (r), the zero profit conditions determine the allocation of innovation between countries, i.e., the temporary equilibrium solution for the technology level (θ) in the rest of the world ($\theta_1 \equiv 1$). Modifying equation (24) to reflect expectations, we obtain two technology arbitrage conditions

$$g_{1t}^e = \left[1 + r_t - \Omega \widehat{k}_1(\theta,\tau) r_t^{\frac{\alpha}{\alpha-1}} \left(\Gamma'(g_{1t}^e - 1)\right)^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}} \equiv F_1(g_{1t}^e, r, \theta, \tau), \tag{41}$$

$$g_{2t}^e = \left[1 + r_t - \Omega \widehat{k}_2(\theta,\tau) r_t^{\frac{\alpha}{\alpha-1}} \left(\Gamma'(g_{2t}^e - 1)\right)^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}} \equiv F_2(g_{2t}^e, r, \theta, \tau). \tag{42}$$

In (41)-(42), the country-specific growth multipliers $\widehat{k}_i$ are equal only if a steady state has been attained or if expectations are symmetric ($g_{1t}^e = g_{2t}^e$).[36]

Lemma 9 *Given growth expectations (g_{1t}^e, g_{2t}^e) and trade policy of each representative country, equations (41)-(42) yield a unique temporary equilibrium solution for the interest rate, $r_t(g_{1t}^e, g_{2t}^e, \tau)$ and the technology level, $\theta(g_{1t}^e, g_{2t}^e, \tau)$.*

Consistent with the temporary value of the interest rate and using equation

$$g_t = [\beta(1 + r_t(g_{1t}^e, g_{2t}^e, \tau)]^{1/\sigma} \equiv T(g_{1t}^e, g_{2t}^e, \tau), \tag{43}$$

consumers choose how much to consume and the amount of consumption to forego; this determines the rate of growth for innovation and the capital stock

[36] The proof of Lemma 9 is in Appendix 4.

at a temporary equilibrium in time period t, namely g_t. It is by combining the behavior of producers and consumers as specified in (41)-(42) and (43) that we obtain the mapping $T : (g^e_{1t}, g^e_{2t}) \to g_t$ from expectations to realized growth at any time period.

Next, we specify the learning rule

$$g^e_{i(t+1)} = g^e_{it} + \gamma_{it}(g_t - g^e_{it}), \quad i = 1, 2. \tag{44}$$

By (44), all decisionmakers adjust previous expectations (g^e_{it}) by an additive correction term that reflects the observed error in expectations. The gain parameters, γ_{it}, that indicate responsiveness of expectations to errors are allowed to vary by individual and over time (heterogeneity in expectations).[37] We assume that the gain sequences satisfy the regularity assumption (Assumption A) in Honkapohja and Mitra (2006).[38] One example of a gain sequence of this kind is $\gamma_{it} = K_i/t$ which, for $K_i = 1$, corresponds to forecasting future growth by its past average rate.

Equations (44) imply that each country group applies similar learning rules but that there may differences in the extent to which expectations are adjusted to mistakes. This assumption is supported by Pfajfar and Santoro (2007) according to whom individuals differ in their ability to forecast economic conditions and may adjust expectations at significantly different speeds. According to Pfajfar and Santoro, the ability to forecast and adjust to errors is positively related to the levels of income and educational achievement. Additional learning heterogeneity can arise if countries hold different initial beliefs about their potential growth; such differences in initial expectations can reflect, for example, different autarky steady state solutions. However, even if the initial expectations were the same for all countries, differences in the gain parameters introduce variations in expectations in subsequent time periods.

Together, equations (43) and (44) define the model of heterogenous learning dynamics. Substituting the T-mapping (43) into the learning rule (44) yields the system

$$g^e_{1(t+1)} = g^e_{1t} + \gamma_{1t}(T(g^e_{1t}, g^e_{2t}, \tau) - g^e_{1t}), \tag{45}$$

$$g^e_{2(t+1)} = g^e_{2t} + \gamma_{2t}(T(g^e_{1t}, g^e_{2t}, \tau) - g^e_{2t}), \tag{46}$$

for which the long run equilibria are the fixed points. Stability of steady states under adaptive learning is determined by local stability of the system (45)-(46) near each fixed point.

Proposition 10 *A steady state equilibrium, (r^*, g^*_Z), that solves equations (22)-(25), is stable under adaptive learning dynamics (as defined in (45)-(46)) if*

$$\mathcal{B} \equiv \left(\frac{\partial r_t(g^*, g^*)}{\partial g^e_{1t}} + \frac{\partial r_t(g^*, g^*)}{\partial g^e_{2t}} \right) - \frac{\partial r_{cons}(g^*)}{\partial g_t} < 0. \tag{47}$$

[37] The gain sequences could also include stochastic variation and random inertia; such extensions would not alter Proposition 10 below.

[38] These regularity conditions require that $\gamma_{i,t} \leq K_i \gamma_t$ for some constant $K_i > 0$, where γ_t satisfies $\sum_{t=1}^{\infty} \gamma_t = \infty$, $\sum_{t=1}^{\infty} \gamma_t^2 < \infty$, and $\limsup(1/\gamma_{t+1} - 1/\gamma_t) < \infty$.

In (47), the partial derivatives $\partial r_t(g^*, g^*)/\partial g^e_{1t}$ and $\partial r_t(g^*, g^*)/\partial g^e_{2t}$ are obtained using the technology arbitrage conditions (41)-(42) and the expression for $\partial r_{cons}(g^*)/\partial g_t$ follows using (43). A steady state equilibrium is unstable if $\mathcal{B} > 0$.

In order to interpret and illustrate stability condition (47) it is helpful to first consider the corresponding condition under the special assumption of *homogenous learning dynamics* ($g^e_{1t} = g^e_{2t}$ and $\gamma_{1t} = \gamma_{2t}$ for all t). In this case, the technology arbitrage conditions (41)-(42) imply

$$\widehat{k}_1(\theta, \tau) = \widehat{k}_2(\theta, \tau) \tag{48}$$

for each time period, and substitution of the $\theta(\tau)$ solution obtained from (48) into (41) yields the common technology arbitrage equation that replaces (41)-(42):

$$g^e_t = \left[1 + r_t - \Omega \widehat{k}_1(\theta(\tau), \tau) r_t^{\frac{\alpha}{\alpha-1}} (\Gamma'(g^e_t - 1))^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}} \equiv F(g^e_t, r, \tau). \tag{49}$$

Combining this equation with (43) and the common learning rule (similar to (45)) yields the relevant stability condition.

Corollary 11 *Under homogenous learning dynamics, a steady state is stable if*

$$\mathcal{B} \equiv \frac{\partial r_t(g^*)}{\partial g^e_t} - \frac{\partial r_{cons}(g^*)}{\partial g_t} < 0, \tag{50}$$

where $\partial r_t(g^*)/\partial g^e_t$ is obtained using (49) and $\partial r_{cons}(g^*)/\partial g_t$ from equation (43). A steady state equilibrium is unstable if $\mathcal{B} > 0$.

For the interpretation of condition (50) we may apply the CC and TT curves of Figure 1. The CC curve represents equilibrium condition (22) which now corresponds to equation (43) (with $g^e_{1t} = g^e_{2t}$); thus, derivative $\partial r_{cons}(g^*)/\partial g_t$ in (50) gives the slope of the CC curve at a steady state in Figure 1. The TT curve depicts equations (23)-(25) which are now replaced by (48) and (49); thus, derivative $\partial r_t(g^*)/\partial g^e_t$ in (50) equals the slope of the TT curve at a steady state. By condition (50), *homogenous learning dynamics are stable near a steady state if the TT curve cuts the CC curve from above at a steady state* in Figure 1 (e.g., equilibria $E1$ and $E3$ in Figure 1 are stable under homogenous adaptive learning, whereas $E2$ is unstable).[39]

[39] An example of stable homogenous dynamics in Figure 1 can be obtained by considering an arbitrary (common) growth expectation near equilibrium $E3$, say. Given the common g^e_t, the temporary equilibrium value of the interest rate, r_t, is read off the common TT curve (this corresponds to solving r_t from (49)). Once r_t is determined, the growth rate at the temporary equilibrium (g_t) is obtained using the CC curve (this corresponds to substituting r_t into (43)). Next period (common) expectations (g^e_{t+1}) are then derived using (44). Condition (50) that is satisfied at $E3$ guarantees that the sequence of temporary equilibria approaches $E3$ over time.

When learning dynamics are not homogenous, stability condition (47) cannot be directly interpreted in terms of Figure 1. This is because in the case of multiple heterogenous learners, a single TT curve is not sufficient to represent the short run adjustment of innovating producers. Even with heterogenous learners, a single TT curve is, however, appropriate for comparisons of long run steady state solutions (in case of multiple equilibria) and for determining the direction of changes in long run growth and interest rate in case of other exogenous changes (shift in the TT curve). Figure 2 illustrates the short run adjustment.

The first and third quadrant of Figure 2 show two country-specific short run TT curves, denoted by $TT_1(\theta_0)$ and $TT_2(\theta_0)$, and these apply to countries 1 and 2, given the initial value of the technology ratio, θ_0 (the TT curves are obtained from equations (41)-(42)); curves $TT_1(\theta_0)$ and $TT_2(\theta_0)$ do not generally coincide because the growth multipliers $\widehat{k}_1(\theta_0, \tau)$ and $\widehat{k}_2(\theta_0, \tau)$ are not necessarily equal when learning is heterogenous. Starting from some initial expectations, e.g., g^e_{1t} and g^e_{2t} in Figure 2, the two TT curves together determine the temporary equilibrium interest rate, r_t, as well as the temporary technology ratio, θ_t.

In Figure 2, given growth expectations g^e_{1t} and the initial θ_0, innovators in country 1 expect to earn a return shown by point b on the 45°-degree line in the second quadrant of the figure. Producers in country 2 expect to earn a return shown by point a. The temporary equilibrium return, r_t, that is the same in all locations is found along the 45°-degree line between the points a and b and is depicted by a dotted line in the first and second quadrant of Figure 2. The short run adjustment in the allocation of innovation (technology level) that takes place at the same time as the investment returns equalize is shown as shifts in the two short run TT curves to positions $TT_1(\theta_t)$ and $TT_2(\theta_t)$: the value of θ_t adjusts so as to attain the common return, r_t, in each country, given g^e_{1t} and g^e_{2t}.

Because the temporary interest rate equalizes, the two short run TT curves shift in opposite directions. Specifically, the TT curve that yields the lower expected return shifts up and the other TT curve shifts down (in Figure 2, curve $TT_2(\theta_0)$ yields point a and so curve TT_2 shifts up; curve TT_1 that yields point b shifts down). The corresponding short run change in the value of the technology level is obtained using the growth multipliers $\widehat{k}_1(\theta, \tau)$ and $\widehat{k}_2(\theta, \tau)$ in (41)-(42), i.e., θ_t increases (declines) if curve TT_1 shifts up (down) (see Appendix 4). Thus, in Figure 2, θ_t declines as a new temporary equilibrium that corresponds to the given expectations is attained.

Once the interest rate at the temporary equilibrium has been determined as a part of the adjustment in Figure 2, substitution of this r_t solution into the consumers' CC curve (43) yields the realized growth rate, g_t, at the temporary equilibrium. Learning rules (45)-(46) then generate the next period expectations, $g^e_{1(t+1)}$ and $g^e_{2(t+1)}$.[40]

We can now better see the meaning of the stability condition (47). As in the case of homogenous learning dynamics, derivative $\partial r_{cons}(g^*)/\partial g$ in (47)

[40] Figure 3 below gives an example of the complete short run adjustment in (g_Z, r)-space.

represents the slope of a CC curve such as shown in Figure 1. This slope indicates the magnitude of consumer saving responses to changes in the interest rate and the degree to which these responses affect the temporary growth rate (given $\triangle r_t$, what is $\triangle g_t$?). The first bracketed term of (47), on the other hand, differs from the slope of the TT curve, $\partial r_t(g^*)/\partial g_t^e$, that appears in condition (50): instead of showing the response in the temporary return to investment as commonly held expectations vary, the first term of (47) gives the aggregate response in r_t as growth expectations vary, taking into account the heterogenous expectations of all locations and the adjustment in the relative technology level (θ_t) that is a part of the producers' short run response as expectations change (given all $\triangle g_{it}^e$, what is $\triangle r_t$?). Altogether, the aggregate derivative is analogous to the slope of a TT curve, but allowing for heterogenous expectations and learning; given this interpretation, both conditions (47) and (50) require that, for stability, a producer derivative that represents the response of investment returns to growth expectations (slope of a TT curve) must be smaller than the aggregate consumer response to the interest rate at the steady state (slope of a CC curve).

When learning is homogenous, the long run TT curve of Figure 1 can be used to illustrate short run learning dynamics (see footnote 39 for an example) but, in the case of heterogenous learning, short run dynamics cannot be as easily described. This occurs because, under homogenous learning, the technology level (θ_t) is independent of the common growth expectations and attains its steady state value during the adjustment of r_t and g_t (so that a single TT curve suffices for the description of short run dynamics); however, when learning is heterogenous, the technology level is a function of growth expectations and, as shown in Figure 2, both r_t and θ_t adjust in short run (so that a single TT curve does not describe short run dynamics).[41]

In general, stability conditions under heterogenous learning include agent-specific as well as aggregate restrictions on behavior (Honkapohja and Mitra (2006)). Above, condition (47) does not restrict partial derivatives $\partial r_t(g^*, g^*)/\partial g_{1t}^e$ and $\partial r_t(g^*, g^*)/\partial g_{2t}^e$ individually because both derivatives are of the same sign (positive), i.e., changes in growth expectations have a qualitatively similar impact on the (common) interest rate. This implies that, in the present model, there is no possibility of instability arising from qualitatively different behaviors on the part of the two types of decision makers (opposite signs for these derivatives) and so an aggregate condition, such as (47), is sufficient for stability of a steady state.

[41] The long run TT curve is obtained using equations (23)-(25). When learning is homogenous, equation (48) that is identical to (25) determines the value of $\theta_t(\tau)$ so that (48) and (49) coincide with (23)-(25); a TT curve such as in Figure 1 can thus be drawn. When learning is heterogenous, equations (41)-(42) do not imply (48). If depreciation is included in the model, θ_t is a function of growth expectations even when learning is homogenous.

5.2 Short Run Adjustment and Growth:

The nature of the short run adjustment process from one steady state toward another is also of interest. While the creation of a customs union slows down growth and reduces the level of innovation in the rest of the world in the long run, there may be systematic short run asymmetries in the growth performance of the two country groups. Such asymmetries can arise from heterogenity in structural parameters and learning dynamics. In the present section, we consider the impact of asymmetries in tariff policy and learning dynamics (the effect of other structural asymmetries will be considered later).

5.2.1 Tariff Reduction Example:

Figure 3a gives an example of the short run adjustment following a *reduction* in the tariff imposed by the customs union (country 1). The initial steady state occurs at (r_0, g_0) and $\tau_0 > 1$ is assumed. Curve CC is analogous to the corresponding curve in Figure 1 and curve $TT(\theta_0, \tau_0)$ represents equilibrium conditions (23)-(25) at a steady state (after full adjustment of expectations and technology level, given τ_0). In the long run, a decrease in the union tariff is expansionary and so the subsequent steady state will occur to the right of point (r_0, g_0) along curve CC and the technology level variable θ_t will also increase ($\theta_0 < 1$ at (r_0, g_0) so that the customs union is the initially more technologically advanced country group).

Curves $TT_1(\theta_0, \tau)$ and $TT_2(\theta_0, \tau)$ in Figure 3a show the immediate (period t) effect of the tariff reduction on the profitability calculations of innovators in countries 1 and 2 (outside the customs union), keeping technology level fixed at θ_0 (these curves are obtained using (41)-(42) with tariff τ taking its post-reform value).[42] Initial expectations in country 1 and 2 equal $g^e_{1t} > g_0$ and $g^e_{2t} = g_0$ as shown.[43]

Given g^e_{2t}, the expected return for investment in country 2, r_{2t}, is read off curve $TT_2(\theta_0, \tau)$ and similarly, given g^e_{1t}, r_{1t} is obtained using $TT_1(\theta_0, \tau)$. Assuming that the equalized rate of return at a temporary equilibrium, r_t, takes the value shown in the figure, the two short run TT curves shift to positions $TT_1(\theta_t, \tau)$ and $TT_2(\theta_t, \tau)$ and thus $\theta_t > \theta_0$, i.e., immediately following the tariff reduction, the relative technology level of countries in the rest of the world increases. In the next time period, after learning takes place, expectations equal $g^e_{1(t+1)}$ and $g^e_{2(t+1)}$ and curves $TT_1(\theta_t, \tau)$ and $TT_2(\theta_t, \tau)$ yield the expected returns for investment, $r_{1(t+1)}$ and $r_{2(t+1)}$, shown by points b and a. The temporary equilibrium in period $(t+1)$ then features $r_{t+1} > r_t$, $g_{t+1} > g_t$, and $\theta_{t+1} > \theta_t$ (technology level of the rest of the world further increases in period $(t+1)$).

Importantly, during the process of adjustment in the temporary equilibrium

[42] Curve $TT_2(\theta_0, \tau)$ lies above $TT_1(\theta_0, \tau)$ because $\widehat{k}_2(\theta_0, \tau) > \widehat{k}_1(\theta_0, \tau)$ (see (35) and (36)).

[43] It may appear more natural to set $g^e_{2t} = g^e_{1t}$ following a tariff reduction surprise. However, given heterogeneity in learning, growth expectations will deviate in the next time period; Figure 3 avoids the additional step by assuming that expectations differ in period t.

the *rates of growth for innovation (and capital stock) are not the same in all countries.* This is because, by construction, the short run growth rate, g_t, here refers to growth in the customs union (country 1), whereas short run growth in innovation outside the customs union includes changes in the value of θ_t over time. In the present example, we observe $\theta_{t+1} > \theta_t > \theta_0$ so that, over time periods t and $(t+1)$ at least, innovation advances faster in the rest of the world than in the customs union, i.e., there are *two growth clubs (* the customs union and the rest of the world) and within each club, short run growth experience is shared but, across groups, short term growth performance varies (in Figure 3a, the rest of the world continues to catch up with the previously more advanced customs union).

The length of time that growth differences persist between growth clubs depends in a complicated manner on learning dynamics and the shifs in innovators' profit calculations that occur as the aggregate economy transitions toward a new steady state. In Figure 3a, the short run growth advantage of the rest of the world lasts for at least two time periods because firms outside the customs union expect a large improvement in profitability as tariffs are reduced (r_{2t} is very high, given τ and g^e_{2t}) and their expectations for future growth sufficiently respond to a positive growth surprise in period t so as to maintain the growth momentum. If learning dynamics in the rest of the world were slower (the gain parameter γ_{2t} low), the same outcome might not occur. If, for example, expectations in country 2 exhibit complete inertia ($g^e_{2(t+1)} = g^e_{2t} = g_0$), then in Figure 3, $r_{2(t+1)} = r_t < r_{1(t+1)}$ and then $\theta_{t+1} < \theta_t$, i.e., the first period of faster technological growth in the rest of the world is interrupted by at least one period of slow growth during which the technological lead of the customs union again widens. Such slow growth could last several time periods if the adjustment in expectations is sufficiently asymmetric in the two country groups.

Periods of relatively slow growth can also arise because of changes in the producer's calculations regarding expected returns. For example, if TT curves slope downwards indicating larger importance of the cost of capital in profit projections, then, following a period of relatively fast growth in the rest of the world ($\theta_t > \theta_0$), the expected return for investment in the customs union may turn out to be higher ($r_{1(t+1)} > r_{21(t+1)}$) in which case $\theta_{t+1} < \theta_t$ is observed (the customs union expands its technological lead in period $(t+1)$ despite the initial reduction in the union tariff).

Further, the immediate reaction to a tariff reduction in the customs union may be a slowdown (not an increase) in innovation in the rest of the world. The initial growth expectations in Figure 3b are similar to those in Figure 3a, but the relative increase in expected returns to investment in country 2 is smaller (curve $TT_2(\theta_0, \tau)$ is located above, but rather close to, curve $TT_1(\theta_0, \tau)$). Then, given g^e_{1t} and g^e_{2t}, $r_{1t} > r_{2t}$ and so the temporary equilibrium in period t is such that $\theta_t < \theta_0$, i.e., the pace of innovation in the customs union in period t exceeds that of the rest of the world. Expressions for the growth multipliers $\widehat{k}_1$ and $\widehat{k}_2$ in (41)-(42) that determine the magnitude of the shifts in $TT_1(\theta_0, \tau)$ and $TT_2(\theta_0, \tau)$ suggest that the outcome of Figure 3c is more likely if the initial tariff is low so that the initial technological lead of the customs union is not very wide. Conversely, Figure 3a may be more likely when the initial tariff of the customs union is high and the initial technology level of the rest of the world is low; then, a reduction of the union tariff is more likely to sufficiently raise the expected return to investment in the rest of the world so that the short run rate of growth outside the customs union actually increases.

Since changes in multipliers $\widehat{k}_1$ and $\widehat{k}_2$ also depend on other structural asymmetries ($\widehat{L}_i$ and υ_i), shifts in curves $TT_1(\theta_0, \tau)$ and $TT_2(\theta_0, \tau)$ and therefore the short run growth dynamics are affected by such asymmetries. E.g., if the rest of the world is subject to adverse asymmetries ($\widehat{L}_2$ low and/or υ_2 high), potential innovators may not perceive a tariff reduction equally positively as they otherwise may; an outcome such as in Figure 3b (persistently slow growth in the rest of the world even after the tariff is lowered) may then be more likely. More advantageous structural parameters, on the other hand, are likely to improve innovators' ability to take advantage of a tariff reduction and the associated improvement in growth expectations; then, the outcome of Figure 3a (immediate faster growth in the rest of the world following the tariff reduction) may be more likely.[44]

5.2.2 Growth in the Short Run:

Several observations follow using Figures 3. First,

- *even if countries experience a common rate of growth at each long run steady state, their growth experiences in the short run need not be the same*

and, second,

- *transition dynamics do not necessarily yield a monotonic sequence of changes toward a new steady state.*

These conclusions reflect the adjustment in technology levels that is a part of the dynamic transition and gives raise to the short run variations in growth country by country. As Figures 3 demonstrate, growth clubs may well appear but their relative performance may oscillate over time even if the long run growth consequences of an exogenous event were clear (above, country 2 will attain a higher relative level of technology in the long run but, in the short run, growth in country 2 may be faster or slower than in the customs union and the relative growth positions may change over time).

Third,

- *heterogeneities in structural parameters (here, tariff policy) are a cause of both long run income clubs and short run growth clubs.*

Above, asymmetry in tariff policy identifies the members of the customs union and the rest of the world, and these policy induced country groups form

[44] Effects of structural asymemtries on short dynamics will be discussed in later versions.

the subsequent income and growth clubs. In the long run, the customs union gains in terms of technology share while possibly losing in relative aggregate output (income clubs) and, in the short run, the tariff separates the two growth clubs for which differences in growth rates may not reflect the expected long run impact of the initial exogenous change. The magnitude of the differences in structural parameters (here, the height of the tariff wall) have an effect on relative outcomes of growth clubs.

Further, because of the transitory nature of growth clubs,

- *there is no fixed correlation between the technology levels of growth clubs and their relative short run growth rates.*

In Figures 3, countries in which the relative level of innovation is initially low (the rest of the world) do not necessarily experience faster growth in the short run and growth may be slower than elsewhere even when a policy change favors this group.

Finally,

- *heterogeneity in learning and initial expectations and can create growth surprises.*

In Figure 3a, slow pace of learning can cause a slow-down in innovation even as long run growth prospects are positive; in Figure 3b, low initial expectations ($g^e_{2t} < g^e_{1t}$) reduce the expected return for investment in the short run ($r_{2t} < r_{1t}$) so that the pace of innovation is slower in the rest of the world than in the customs union.

Further, while we have not explored the connection here, *heterogeneity in learning may be partly a consequence of economic policy.* In particular, if a low level of technological attainment negatively impacs the speed of learning as suggested by empirical evidence (Pfajfar and Santoro (2007)), then inertia in expectations may well reflect the policy induced relative technology position of a country group (e.g., above, the low technology share of the rest of the world following the establishment of the customs union). If, as seems likely, economic forecasting and expectational learning are a production process that requires capital goods, such learning should be easier and less costly in locations that innovate more and include a large share of the world markets (above, a large customs union); learning would be more difficult and slower in a location in which the level of innovation is low (above, the rest of the world).[45]

Analogous conclusions are obtained if a tariff increase is considered (establishment of a customs union or an increase in the union tariff). In the long run, the customs union attains a technological lead position while growth at the long run steady state will be slower. But, in the short run, the customs union is likely to experience a transitory boost in growth relative to the rest of the world (if the temporary equilibria involve a decreasing θ_t sequence) and, depending

[45] Models with dynamic choice of predictors are briefly discussed in Evans and Honkapohja (2001: Ch. 15.5-15.6).

on growth expectations and changes in firms' expected profitability, such asymmetric growth may persist for a considerable time period. Other details of the union policy and any structural asymmetries between the union and the rest of the world may also be important because the shifts in the profitability of investment (the short run TT curves) depend on all such factors.

6 Conclusions:

7 Appendices:

7.1 Appendix 1:

The slope of the TT curve: The slope of the TT curve is obtained by differentiating equations (23) and (24):

$$dg_Z = \mathcal{C}(1-\mathcal{E})dr - \mathcal{C}\mathcal{D}dp^Z - \mathcal{C}\mathcal{F}d\widehat{k}_1, \tag{51}$$

$$dp^Z = \Gamma'' dg_Z, \tag{52}$$

where

$$\mathcal{C} \equiv \left(\frac{1+\xi}{\xi}\right)\left[1+r-\Omega\widehat{k}_1 r^{\frac{\alpha}{\alpha-1}}(p^Z)^{\frac{1}{\alpha-1}}\right]^{\frac{1}{\xi}} \quad (>0), \quad \frac{1+\xi}{\xi} = \frac{\phi-\alpha}{\phi-1}, \tag{53}$$

$$\mathcal{E} \equiv \frac{\widehat{k}_1\alpha\Omega}{\alpha-1} r^{\frac{1}{\alpha-1}}(p^Z)^{\frac{1}{\alpha-1}} \quad (<0), \tag{54}$$

$$\mathcal{D} \equiv \frac{\Omega\widehat{k}_1}{\alpha-1} r^{\frac{\alpha}{\alpha-1}}(p^Z)^{\frac{2-\alpha}{\alpha-1}} \quad (<0), \tag{55}$$

$$\mathcal{F} \equiv \Omega r^{\frac{\alpha}{\alpha-1}}(p^Z)^{\frac{1}{\alpha-1}} \quad (>0). \tag{56}$$

Then,

$$\frac{dr(g_Z)}{dg_Z} = \frac{1+\mathcal{C}\mathcal{D}\Gamma''}{\mathcal{C}(1-\mathcal{E})}. \tag{57}$$

According to (57) and since $\widehat{k}_1$ is positive, the TT curve is upward sloping if $\Gamma'' = 0$ and $\phi > 1$ but can have negative slope of the numerator of (57) is negative. HTR (2002) further discuss the slope of the TT curve in a symmetric two country model.

Proof of Proposition 2: We consider the effect of an increase in $\widehat{k}_1$ on the TT curve in Figure 1 described by (23) and (24). Keeping r fixed, totally differentiating (23) and (24) and using the analogy of this total differential to equations (51) and (52) above, we obtain

$$\frac{dg_Z}{d\widehat{k}_1} = -\frac{\mathcal{C}\mathcal{F}}{1+\mathcal{C}\mathcal{D}\Gamma''}. \tag{58}$$

Expression (57) for the slope of the TT curve implies that the derivative (58) is negative (positive) when the TT curve slopes up (down). In Figure 1, these signs of the derivative (58) correspond to a global upward shift of the TT curve following any increase in the multiplier $\widehat{k}_1$.

If the free trade steady state is unique (and stable), g_Z^F increases as the TT curve shifts up. (If there are multiple equilibria, Section 5 yields that equilibria at which the TT curve cuts the CC curve from above are stable and, at such equilibria, g_Z^F increases as the TT curve shifts up.)■

7.2 Appendix 2:

Proof of Proposition 3: Equation (34) yields

$$\left(\frac{n_1 + n_2\theta\tau^{\frac{\gamma}{\gamma-1}}}{n_1 + n_2\theta}\right)^{\xi} = \frac{n_2}{n_1}\left(\frac{\theta^{\xi} - 1}{\tau^{\frac{1}{\gamma-1}} - \theta^{\xi}}\right), \quad \theta^{\xi} \neq \tau^{\frac{1}{\gamma-1}}. \tag{59}$$

Because $\tau > 1$, the left-hand side of (59) is positive and less than one. Therefore, the right-hand side of (59) is also positive and less than one. This implies condition (37).

Equation (59) is equivalent to

$$(n_1 + n_2\theta\tau^{\frac{\gamma}{\gamma-1}})^{\xi}(\tau^{\frac{1}{\gamma-1}} - \theta^{\xi}) = \frac{n_2}{n_1}(\theta^{\xi} - 1)(n_1 + n_2\theta)^{\xi}. \tag{60}$$

Given any θ that solves (60), the left-hand side of the equation decreases if τ increases. Therefore, the right-hand side of (60) must also decrease so as to maintain the equation and this occurs if θ decreases. Thus, the solution for θ is a decreasing function of tariff τ.■

The steady state solution for θ decreases if ξ decreases and α and γ are fixed: Equation (59) is equivalent to

$$\frac{n_1 + n_2\theta\tau^{\frac{\gamma}{\gamma-1}}}{n_1 + n_2\theta} = \left(\frac{n_2}{n_1}\right)^{\frac{1}{\xi}}\left(\frac{\theta^{\xi} - 1}{\tau^{\frac{1}{\gamma-1}} - \theta^{\xi}}\right)^{\frac{1}{\xi}}. \tag{61}$$

The first term on the right-hand side of (61) increases as ξ (or ϕ) decreases. For a given value of θ, the derivative of the second term on the right-hand side of (61) equals

$$\frac{d}{d\xi}\left[\left(\frac{1 - \theta^{\xi}}{\theta^{\xi} - \tau^{\frac{1}{\gamma-1}}}\right)^{\frac{1}{\xi}}\right] = \tag{62}$$

$$-(1 - \tau^{\frac{1}{\gamma-1}})\theta^{\xi-1}\left[\frac{1 - \theta^{\xi}}{\theta^{\xi} - \tau^{\frac{1}{\gamma-1}}}\right]^{\frac{1}{\xi}}\left[\frac{1}{(\theta^{\xi} - \tau^{\frac{1}{\gamma-1}})(1 - \theta^{\xi})}\right] < 0.$$

In (62), $-(1 - \tau^{\frac{1}{\gamma-1}})\theta^{\xi-1} < 0$ because $\tau > 1$ and, by (61), the next bracketed term is positive. The sign of the last bracketed term in (62) is determined by (61) as well: the two terms in the denominator must have the same sign because the left-hand side of (61) is positive. Thus, the derivative (62) is negative, i.e., as ξ decreases (ϕ decreases), the second term on the right hand side of (61) increases.

Therefore, if ξ decreases, both terms on the right-hand side of (61) increase, given a fixed value of θ. Accordingly, the left-hand side of (61) must increase. This implies that the θ solution to (61) must decrease if ξ decreases because τ remains fixed.

Proof of Proposition 4: Equation (59) is equivalent to

$$\frac{1}{n_2/n_1}\left(\frac{1+\left(\frac{n_2}{n_1}\right)\theta\tau^{\frac{\gamma}{\gamma-1}}}{1+\left(\frac{n_2}{n_1}\right)\theta}\right)^{\xi}=\frac{\theta^{\xi}-1}{\tau^{\frac{1}{\gamma-1}}-\theta^{\xi}}. \tag{63}$$

For any given θ (< 1), if n_2/n_1 increases, the left-hand side of equation (63) decreases. Thus, the right-hand side of (63) must decrease. The derivative of the right-hand side with respect to θ is negative since $\theta < 1$ and so θ must increase so as to maintain equation (63).■

Proof of Proposition 6: i) The derivative of the ratio (39) with respect to τ equals (ignoring the power $(1+\xi)$ that does not affect the sign of the derivative)

$$\frac{\partial(.)}{\partial\tau}=(n_2\tau^{\frac{\gamma}{\gamma-1}}\frac{d\theta}{d\tau}+\frac{n_2\theta\gamma}{\gamma-1}\tau^{\frac{1}{\gamma-1}})(n_1+n_1\theta)-n_2\frac{d\theta}{d\tau}(n_1+n_2\theta\tau^{\frac{\gamma}{\gamma-1}}) \tag{64}$$

$$=(n_1n_2+n_2^2\theta)\frac{d\theta}{d\tau}(\tau^{\frac{\gamma}{\gamma-1}}-1)+\frac{n_2\theta\gamma}{\gamma-1}\tau^{\frac{1}{\gamma-1}}+\frac{n_2^2\theta^2\gamma}{\gamma-1}\tau^{\frac{1}{\gamma-1}}. \tag{65}$$

The first term in expression (65) is positive because $d\theta/d\tau < 0$ but the remaining two terms are negative. The sign of expression (65) is therefore ambiguous, except at free trade where it is negative. By continuity, the expression remains negative near the free trade equilibrium.

ii) According to (39),

$$\frac{Y_1}{Y_j}=\left(\frac{1+\left(\frac{n_2}{n_1}\right)\theta\tau^{\frac{\gamma}{\gamma-1}}}{1+\left(\frac{n_2}{n_1}\right)\theta}\right)^{1+\xi}. \tag{66}$$

Then, if n_1/n_2 increases, the ratio Y_1/Y_j increases (by Proposition 4, θ decreases when n_1/n_2 increases and $\tau^{\frac{\gamma}{\gamma-1}} < 1$).■

Proof of Proposition 7: Using arguments analogous to the proof of Proposition 2 above, it can be shown that the TT curve in Figure 1 shifts up if the growth multiplier $\widehat{k}_1$ increases. Further, because $\widehat{k}_1 = k_1/\upsilon_1^{\varsigma}$ and k_1 is defined in (35), we have $\widehat{k}_1/d\tau < 0$ for all $\tau \geq 1$. Thus, the growth multiplier monotonically decreases as the union tariff increases.

According to results in Section 5, world steady states at which the TT curve cuts the CC curve from above are stable under adaptive learning. Then, following an increase in the union tariff and the corresponding downward shift of the TT curve, the g_Z solution at the new stable steady state must be lower than at the initial steady state. The reduction in the growth solution is the larger the higher the tariff.■

7.3 Appendix 3:

Derivation of condition (40): Equation (59) in the proof of Proposition 3 is replaced by equation

$$\left(\frac{n_1 + n_2\theta\tau_U^{\frac{\gamma}{\gamma-1}}}{n_1\tau_R^{\frac{\gamma}{\gamma-1}} + n_2\theta}\right)^{\xi} = \frac{n_2}{n_1}\left(\frac{\theta^{\xi}\tau_R^{\frac{1}{\gamma-1}} - 1}{\tau_U^{\frac{1}{\gamma-1}} - \theta^{\xi}}\right), \quad \theta^{\xi} \neq \tau_U^{\frac{1}{\gamma-1}}. \tag{67}$$

We can no longer conclude that the left-hand side of (67) is less than one. However, the left-hand side is positive and, therefore, the right-hand side of (67) is also positive. This gives condition (40).■

Proof of Proposition 8: i) Equation (67) is equivalent to

$$\left(n_1 + n_2\theta\tau_U^{\frac{\gamma}{\gamma-1}}\right)^{\xi}(\tau_U^{\frac{1}{\gamma-1}} - \theta^{\xi}) = \left(\frac{n_2}{n_1}\right)(\theta^{\xi}\tau_R^{\frac{1}{\gamma-1}} - 1)(n_1\tau_R^{\frac{\gamma}{\gamma-1}} + n_2\theta)^{\xi}. \tag{68}$$

Given a θ value that solves (68), an increase in the retaliatory tariff τ_R reduces the right-hand side of (68). Therefore, the left-hand side of equation (68) must also decrease and, given a fixed value of τ_U, θ must adjust. To determine the direction of change in θ, we obtain the derivative of the left-hand side in (68):

$$\frac{d\left[(n_1 + n_2\theta\tau_U^{\frac{\gamma}{\gamma-1}})^{\xi}(\tau_U^{\frac{1}{\gamma-1}} - \theta^{\xi})\right]}{d\theta} = -2n_2\theta^{\xi}\tau_U^{\frac{\gamma}{\gamma-1}} - n_1\theta^{\xi-1} + n_2\tau_U^{\frac{\gamma+1}{\gamma-1}} \tag{69}$$

$$= -n_1\theta^{\xi-1} + n_2\tau_U^{\frac{\gamma}{\gamma-1}}(-2\theta^{\xi} + \tau_U^{\frac{1}{\gamma-1}}) < 0.$$

The inequality follows because $-2\theta^{\xi} + \tau_U^{\frac{1}{\gamma-1}} < 0$, and this follows using (40). Since the derivative (69) is negative, the value of θ that solves (68) must increase as τ_R increases.

ii) Using (27),

$$\frac{\partial(Y_1/Y_j)}{\partial\tau_R} > 0 \Longleftrightarrow \widehat{S}_1 > \widehat{S}_j, \tag{70}$$

where $\widehat{S}_1 \equiv (dS_1/d\tau_R)/S_1$ and $\widehat{S}_j \equiv (dS_j/d\tau_R)/S_j$, respectively. Since S_1 and S_j equal

$$S_1 = n_1 + n_2\theta\tau_U^{\frac{\gamma}{\gamma-1}}, \; S_j = n_1\tau_R^{\frac{\gamma}{\gamma-1}} + n_2\theta, \tag{71}$$

condition (70) is satisfied if and only if

$$n_2(\tau_R^{\frac{\gamma}{\gamma-1}}\tau_U^{\frac{\gamma}{\gamma-1}} - 1)\frac{\partial\theta}{\partial\tau_R} + \frac{\gamma\tau_R^{\frac{1}{\gamma-1}}}{1-\gamma}S_1 > 0. \tag{72}$$

Thus, at free trade ($\tau_U = \tau_R = 1$) and sufficiently near it, expression (72) is positive.

The growth multiplier equals $\widehat{k}_1 = \widehat{L}(n_1 S_1^{\xi} + n_2 S_j^{\xi} \tau_R^{\frac{1}{\gamma-1}})/\upsilon_1^{\varsigma}$ which yields (ignoring $\widehat{L}$ and υ_1)

$$\frac{\partial \widehat{k}_1}{\partial \tau_R} = \xi \left[n_1 S_1^{\xi-1} \frac{\partial S_1}{\partial \tau_R} + n_2 S_j^{\xi-1} \frac{\partial S_j}{\partial \tau_R} \tau_R^{\frac{1}{\gamma-1}} \right] + \frac{n_2 S_j^{\xi} \tau_R^{\frac{2-\gamma}{\gamma-1}}}{\gamma - 1}. \tag{73}$$

Given (71),

$$\frac{\partial S_1}{\partial \tau_R} = n_2 \frac{\partial \theta}{\partial \tau_R} > 0, \; \frac{\partial S_j}{\partial \tau_R} = \frac{n_1 \gamma}{\gamma - 1} \tau_R^{\frac{1}{\gamma-1}} + n_2 \frac{\partial \theta}{\partial \tau_R}. \tag{74}$$

At free trade, $S_1 = S_j = N$ and so (73) yields

$$\frac{\partial \widehat{k}_1}{\partial \tau_R} = \xi N^{\xi-1} n_2 \left[N \frac{\partial \theta}{\partial \tau_R} + \frac{n_1 \gamma}{\gamma - 1} \right] + \frac{n_2 N^{\xi}}{\gamma - 1}. \tag{75}$$

In order to obtain the sign of (75), we must evaluate $\partial\theta/\partial\tau_R$ at free trade. Differentation of equation (67) at free trade yields

$$\frac{\partial \theta}{\partial \tau_R} = \frac{n_2}{N^{\xi}(1-\gamma)} > 0 \tag{76}$$

and substituting (76) into (75) gives

$$\frac{\partial \widehat{k}_1}{\partial \tau_R} = \frac{n_2}{\gamma - 1}(N^{\xi} - \xi n_2) + \frac{n_1 n_2 \xi \gamma N^{\xi-1}}{\gamma - 1} < 0, \tag{77}$$

if $\xi < 1$.■

7.4 Appendix 4:

Proof of Lemma 9: Given (g_{1t}^e, g_{2t}^e) and tariffs of each country, equations (41)-(42) yield a unique temporary equilibrium solution for (r_t, θ_t) because i) equation (41) yields, for a fixed θ, a unique (increasing) solution $r = r_1(\theta)$, ii) equation (42) also yields, for a fixed θ, a unique (increasing) solution $r = r_2(\theta)$, and iii) the equation $r_1(\theta) = r_2(\theta)$ has a unique solution because the slopes of the functions $r_i(\theta)$ differ.■

Proof of Proposition 10: Linearization of equation (43) yields

$$dg_t = \mathcal{A} \left(\frac{\partial r_t(g^*, g^*)}{\partial g_{1t}^e} d\widetilde{g}_t^e + \frac{\partial r_t(g^*, g^*)}{\partial g_{2t}^e} dg_{2t}^e \right) \equiv A_1 dg_{1t}^e + A_2 dg_{2t}^e, \tag{78}$$

where

$$\mathcal{A} \equiv \frac{1}{\sigma} \beta^{\frac{1}{\sigma}} (1 + r^*)^{\frac{1-\sigma}{\sigma}}, \tag{79}$$

and the partial derivatives $\partial r_t(g^*, g^*)/\partial g_{1t}^e$ and $\partial r_t(g^*, g^*)/\partial g_{2t}^e$ are found by totally differentiating equations (41)-(42). The Implicit Function Theorem yields

$$\begin{bmatrix} \frac{dr_t}{dg_{1t}^e} & \frac{d\theta_t}{dg_{1t}^e} \\ \frac{dr_t}{dg_{2t}^e} & \frac{d\theta_t}{dg_{2t}^e} \end{bmatrix} = \frac{1}{\det F} \begin{bmatrix} F_{2\theta} & -F_{1\theta} \\ -F_{2r} & F_{1r} \end{bmatrix} \begin{bmatrix} F_{1g_{1t}^e} & 0 \\ 0 & F_{1g_{2t}^e} \end{bmatrix}, \tag{80}$$

and

$$\frac{1}{\det F} = \frac{1}{F_{1r}F_{2\theta} - F_{1\theta}F_{2r}} \quad (\neq 0). \tag{81}$$

Therefore,

$$\frac{dr_t}{dg_{1t}^e} = \frac{1}{\det F} F_{2\theta} F_{1g_{1t}^e}, \tag{82}$$

$$\frac{dr_t}{d\widehat{g}_t^e} = -\frac{1}{\det F} F_{1\theta} F_{2g_{2t}^e}. \tag{83}$$

The partial derivatives in (82) and (83) equal

$$F_{1g_{1t}^e} = F_{2g_{2t}^e} = 1 + \mathcal{C}\mathcal{D}\Gamma'' \quad (> 0), \tag{84}$$

$$F_{1r} = F_{2r} = \mathcal{C}(1 - \mathcal{E}) \quad (> 0), \tag{85}$$

$$F_{1\theta} = -\mathcal{C}\mathcal{F}\frac{d\widehat{k}_1(\theta)}{d\theta}, \quad F_{2\theta} = -\mathcal{C}\mathcal{F}\frac{d\widehat{k}_2(\theta)}{d\theta}, \tag{86}$$

where the matrices $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, and $\mathcal{F}$ are defined in (53)-(56) and are evaluated at a steady state. Further, since

$$\widehat{k}_1 = \frac{\widehat{L}}{\upsilon^\zeta}\left[n_1(n_1 + n_2\theta\tau^{\frac{\gamma}{\gamma-1}})^\xi + n_2(n_1 + n_2\theta)^\xi\right], \tag{87}$$

$$\widehat{k}_2 = \frac{\widehat{L}\left[n_1(n_1 + n_2\theta\tau^{\frac{\gamma}{\gamma-1}})^\xi \tau^{\frac{1}{\gamma-1}} + n_2(n_1 + n_2\theta)^\xi\right]}{\upsilon^\zeta\theta^\xi}, \tag{88}$$

assuming that the rest of the world does not retaliate (the case with retaliation is analogous), we obtain that

$$\frac{d\widehat{k}_1(\theta)}{d\theta} = \frac{\xi\widehat{L}n_2}{\upsilon^\zeta}\left[n_1(n_1 + n_2\theta\tau^{\frac{\gamma}{\gamma-1}})^{\xi-1}\tau^{\frac{\gamma}{\gamma-1}} + n_2(n_1 + n_2\theta)^{\xi-1}\right] \quad (> 0), \tag{89}$$

$$\frac{d\widehat{k}_2(\theta)}{d\theta} = \frac{\xi\widehat{L}\theta^{-1-\xi}}{\upsilon^\zeta}\left[-n_1^2(n_1 + n_2\theta\tau^{\frac{\gamma}{\gamma-1}})^{\xi-1}\tau^{\frac{1}{\gamma-1}} + n_2(n_1 + n_2\theta)^{\xi-1}(\theta(1 - n_2) - n_1)\right] \quad (< 0). \tag{90}$$

Then, $\det F > 0$ because $F_{1\theta} < 0$, $F_{2\theta} > 0$, and $F_{1r} = F_{2r} > 0$, and

$$\frac{dr_t}{dg_{1t}^e} = -\frac{\mathcal{C}\mathcal{F}(1 + \mathcal{C}\mathcal{D}\Gamma'')}{\det F}\left(\frac{d\widehat{k}_2(\theta)}{d\theta}\right) > 0, \tag{91}$$

$$\frac{dr_t}{dg_{2t}^e} = \frac{\mathcal{C}\mathcal{F}(1 + \mathcal{C}\mathcal{D}\Gamma'')}{\det F}\left(\frac{d\widehat{k}_1(\theta)}{d\theta}\right) > 0. \tag{92}$$

By equations (91) and (92), multipliers A_1 and A_2 in (78) are both positive. Therefore, applying Proposition 9 of Honkapohja and Mitra (2006) we obtain that the aggregate economy is stable for learning rules specified in (44) if

$$1 - A_1 - A_2 > 0, \tag{93}$$

and this condition is equivalent to condition (47).■

The change in the technology level (θ) corresponding to the shift in the TT curves in Figure 2: Given that derivatives (89) and (90) are of opposite sign, the shifts of the short run TT curves must be in the opposite directions so as to identify a common direction of change in θ_t. The positive sign of derivative (89) implies that the value of θ_t increases if TT_1 shifts up.■

8 Figures:

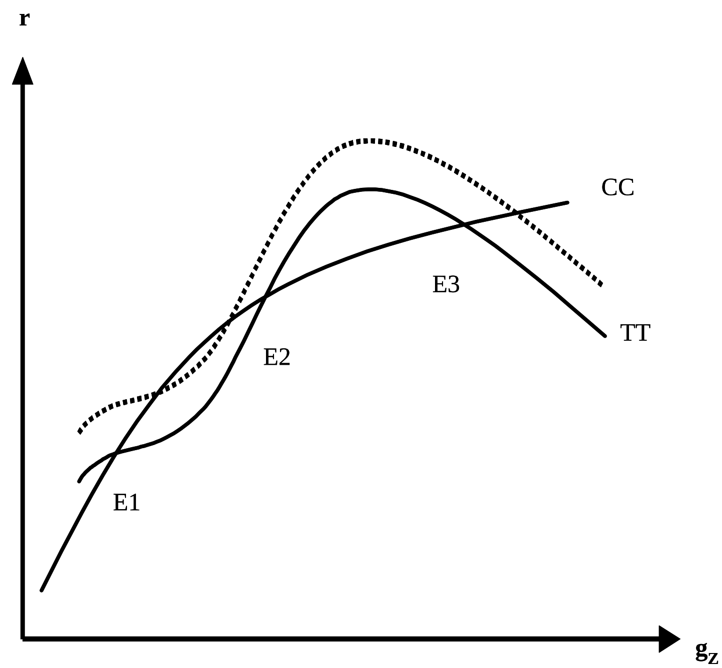

FIGURE 1: Steady States (Long Run Equilibria)

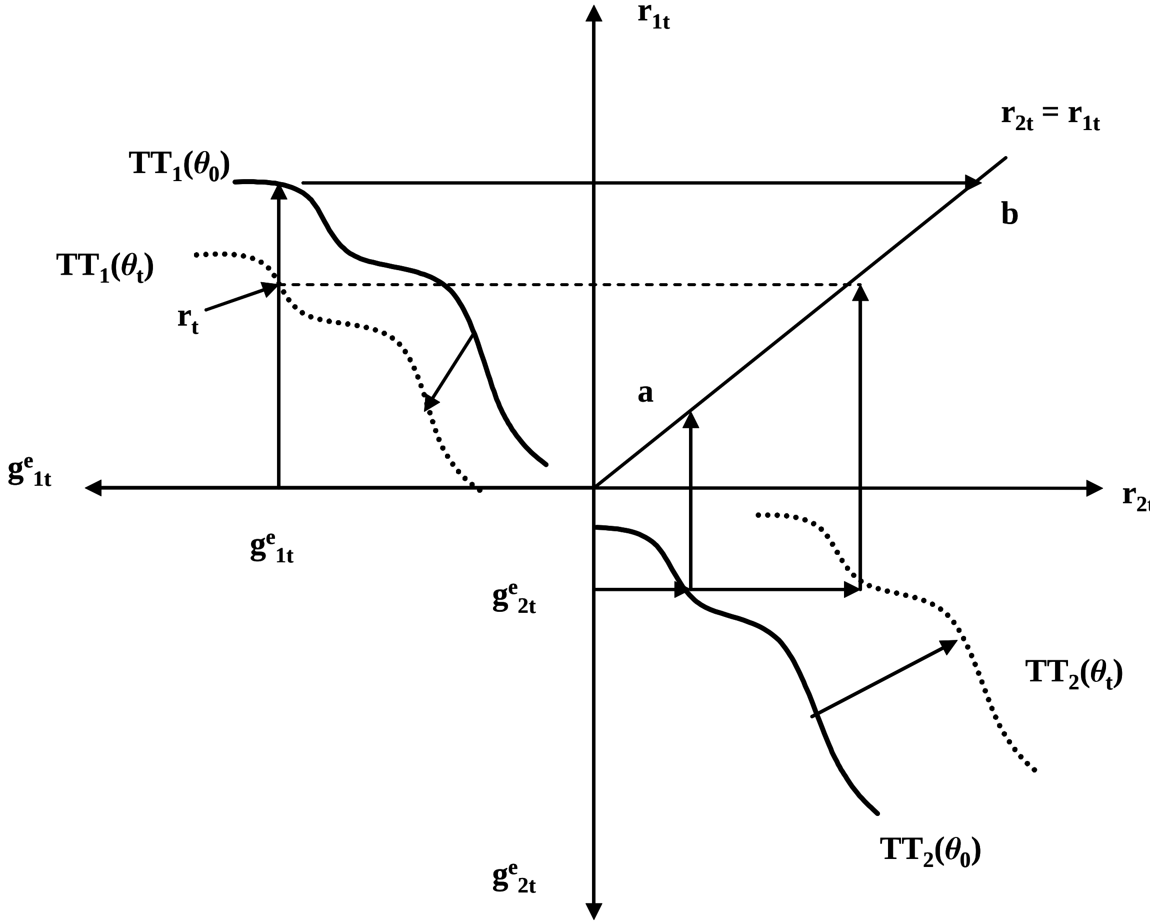

FIGURE 2: Adjustment of the interest rate and technology level

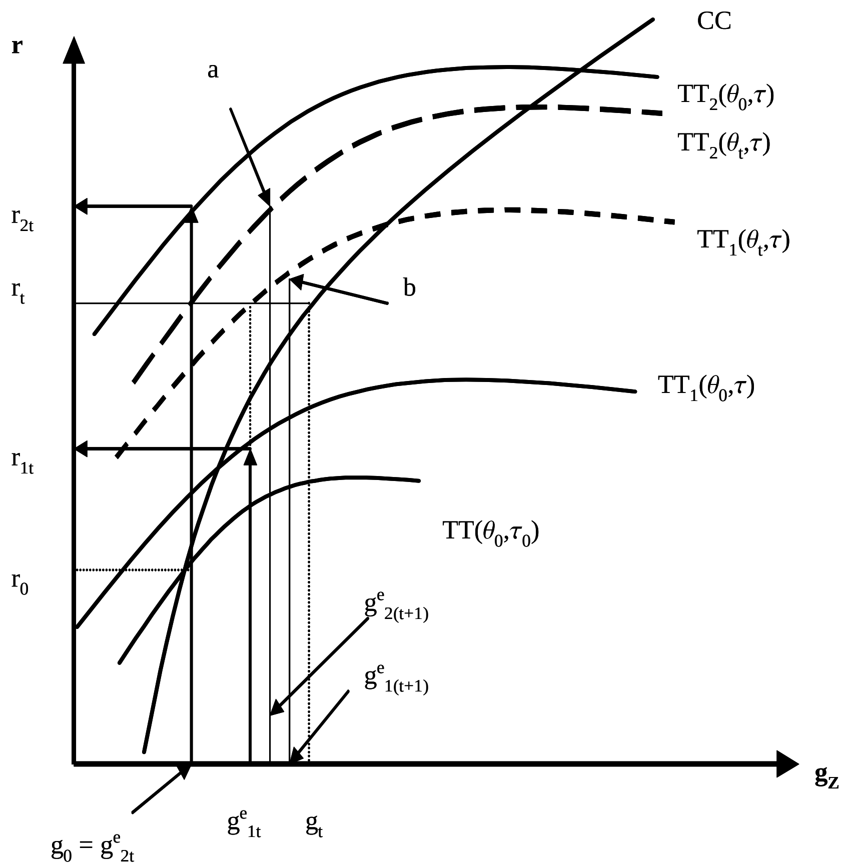

FIGURE 3a: Short Run Adjustment with a Two-Period Growth Club

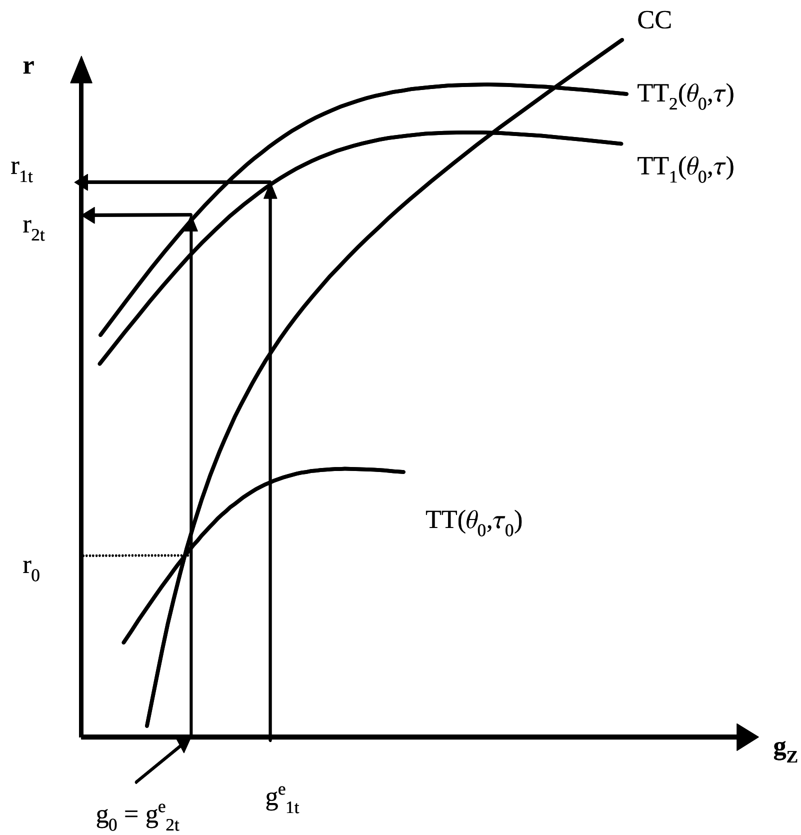

FIGURE 3b: Short Run Adjustment

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