# An extended Ricardian model under the assumption that consumption requires labour effort[*]

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## Abstract

The principle of comparative advantage is typically first explained with the aid of a simple Ricardian model involving two countries, two commodities and one factor. In such a model it is implicitly assumed that consumption is instantaneous or economic agents have sufficient time to consume any bundle of goods. The present paper extends the traditional approach by incorporating a Gossenian–Beckerian consumption time constraint to the Ricardian model of trade. Not surprisingly, the autarkic equilibrium 'full' price ratio is shown to lie strictly between the production and consumption opportunity costs. In response to an exogenous improvement in the consumption technology of one good, the range of the feasible amount of labour allocated to production expands but the amount of labour time devoted to production may increase, decrease or remain unchanged. While the cherished principle of comparative advantage continues to hold in the extended model, the consumption gain from trade is zero and the specialization gain can be negative. In a post-trade equilibrium, the range of the feasible amount of labour allocated to production contracts and a trading nation always devotes less labour time to production.

*Keywords*: Ricardian trade model; time constraint; comparative advantage

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## 1. Introduction

In trade textbooks, the principle of comparative advantage is usually first expounded with the aid of a simple Ricardian trade model consisting of two countries, two goods and one factor of production (homogenous labour). Under the assumptions of fixed labour endowment and constant returns to scale technology, each country's production possibility locus is well defined. A country's offer curve can then be generated by adding a social indifference map to its production possibility locus. Under further assumptions (like the homotheticity of preferences), the two countries' offer curves intersect uniquely between the two autarkic relative price ratios. This world equilibrium completely determines each country's production, trade pattern and consumption.

This kind of approach emphasizes the production tradeoff between commodities and implicitly assumes either that consumption is instantaneous or that a typical economic agent has a sufficiently large amount of time to consume any bundle of commodities. Neither of these assumptions is plausible. Consumption does take time and every agent has a finite amount of time to allocate between various activities, including work and consumption. The importance of the time constraint was emphasized by Gossen (1854), whose insight was praised by pioneering neoclassical theorists but conveniently ignored in their work and in that of subsequent mainstream economists.[1] However, in recent times, his work has been discussed in depth by Georgescu–Roegen (1983, 1985), Niehans (1990), Steedman (2002) and Kemp (2008).

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[1] A historical summary of this neglect is provided by Steedman (2002, chapter 2).

A more general theory of time allocation was developed in the mid 1960s by Becker (1965). His approach emphasized the role of utility maximizing households as productive agents which combine time and market goods via household production functions to generate vectors of basic commodities that enter directly into household utility functions. This can be viewed as a generalization of Gossen's idea.[2] Becker's integration of household production and consumption differs fundamentally from the textbook distinction between households as consumption units and firms as production units.

The primary purpose of this paper is to investigate the impact of Gossenian–Beckerian time constraints on a simple, static Ricardian trade model. Section 2 examines the equilibrium of an autarkic Ricardian economy. Section 3 analyzes the effect of trade on a small Ricardian economy whereas Section 4 briefly considers how the world trade equilibrium is determined. The final section summarizes the main results obtained.


## 2. An autarkic Ricardian economy


Consider first a static, closed economy populated by households which are identical in all respects (including, for example, preferences, labour endowment, access to information, etc).[3] The only resource in the economy is homogeneous

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[2] Interestingly enough, Becker did not refer to Gossen's insight in this often cited work. Not surprisingly, the subsequent literature on the incidence of commodity taxes when consumption is time consuming does not refer to Gossen either (see, for example, Gahvari 2007).
[3] The assumption of representative agents is not often made explicit in expositions of the Ricardian model of trade. As long as economic agents know that they are identical, they will cooperate so that the population behaves like a single individual.

labour (measured in time units) which can be alternatively allocated between production and consumption. For each agent, consumption and production cannot be undertaken simultaneously and the total amount of time available for consumption and production is equal to 24 hours a day minus sleeping time. By choosing the unit of measurement appropriately, we can normalize the total amount of labour available to unity.

The economy produces two private, perishable goods (X and Y) by means of labour under constant returns to scale. Let $a_X$ and $a_Y$ represent the fixed labour per unit of output in the X and Y industries, respectively, and let L be the total amount of labour devoted to production. Then the aggregate production possibility locus can be expressed as

$$a_X X + a_Y Y = L, 0 \leq L \leq 1. \tag{1}$$

The consumption of each commodity takes time. For simplicity, the rates of consumption per unit of time are assumed to remain constant for both goods. Let $b_X$ and $b_Y$ represent the fixed amounts of time required to consume one unit of goods X and Y, respectively. While many goods can be consumed by a single consumer alone, other types of consumption necessarily require two or more persons (e.g., a soccer game).[4] The latter types of consumption complicate the time constraint, and are assumed away in this paper. Under this simplified scenario, the aggregate consumption time constraint can be expressed algebraically as[5]

$$b_X X + b_Y Y = 1 - L, 0 \leq L \leq 1. \tag{2}$$

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[4] Before any interaction can take place, there is also the problem of negotiating the pooling of time. This problem is ignored in the present paper.
[5] Note that, under autarky, the symbols X and Y can be used for production and consumption interchangeably without creating confusion.

Before proceeding further, it is important to note that (1) and (2) are aggregative relationships that do not hold at the individual level. This is because individuals cannot produce and consume simultaneously. To escape the difficulty of producing and consuming at constant steady rates, as indicated by (1) and (2), a credit market may be introduced. This would allow each individual to produce and consume sequentially while aggregate consumption and production remain steady. Alternatively, the difficulty could be avoided by assuming that all households are large enough to have some members working and others consuming at each moment.

The model is completed by specifying the social utility function. Since households are identical in all respects it is possible to speak of a social utility function. It is true that households may derive pleasure not only from the goods consumed but also from interpersonal interaction in joint consumption (e.g., dining with friends can be more enjoyable than dining alone). For the sake of simplicity the psychic satisfaction from consumers' interaction in interpersonally joint consumption is assumed away. We can then summarize the social preferences by a conventional utility function $U(X, Y)$ where $U$ is supposed to be twice differentiable, strictly increasing and strictly concave in its two arguments and to satisfy the regularity assumption that the marginal rate of substitution of $X$ for $Y$ approaches infinity (zero) as $X$ approaches zero (infinity).[6]

The economy's problem in autarky is to maximize $U(X, Y)$ by the choice of $L$, $X$ and $Y$ subject to (1) and (2).[7] To avoid a trivial solution, we shall also assume that

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[6] This regularity assumption will ensure that the country consumes both goods in equilibrium.
[7] This paper approaches the problem from an aggregative perspective. A micro-based alternative is to examine the consumer utility maximization problem (in which a typical household allocates its total time between consumption and working to maximize its utility, taking prices and wage rate as given)

$a_Y/a_X \neq b_Y/b_X$ [8]  Defining the Lagrangean, deriving the first order conditions for an interior equilibrium and eliminating the Lagrangean multipliers, the economy's autarkic equilibrium $(L^o, X^o, Y^o)$ can be uniquely characterized by (1), (2) and

$$U_X(X, Y)/U_Y(X, Y) = (a_X+b_X)/(a_Y+b_Y) \tag{3}$$

where $U_X$ and $U_Y$ stand for the marginal utilities with respect to $X$ and $Y$, respectively.

Conditions (1) and (2) together imply that the autarkic equilibrium occurs at an intersection of the aggregate production possibility and consumption time loci. Thus, we may state:

**Proposition 1**: The country's autarkic equilibrium, characterized by (1), (2) and (3), exists uniquely. In the autarkic equilibrium both production and consumption time constraints are binding and the equilibrium point occurs at an intersection between the aggregate production possibility and consumption time loci.

Consider an intersection of aggregate production possibility and consumption time loci at a feasible $L$. The equation for the locus of all such intersections can be obtained by adding (1) and (2):

$$(a_X+b_X)X + (a_Y+b_Y)Y = 1 \tag{4}$$

Equation (4) can be thought of as the effective budget facing the economy where, as suggested by Becker, the coefficient $(a_i+b_i)$ can be interpreted as the full price of the $i$-th good $(i = X, Y)$, i.e., the sum of the direct price of the $i$-th good and of the time use per unit consumption of the $i$-th good.

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and the firm profit maximization problem (in which a perfectly competitive firm hires labour to produce and seeks to maximize its profits). The standard consumer's tangency condition and the firm's zero profit conditions together produce the equilibrium condition (3) in this paper.

[8] If $a_Y/a_X = b_Y/b_X$ then it is not difficult to show that the aggregate production possibility and consumption time loci coincide in equilibrium and the socially optimal $L = a_X/(a_X+b_X) = a_Y/(a_Y+b_Y)$.

Condition (3) implies that the autarkic equilibrium can also be derived by maximizing $U(X, Y)$ by the choice of $X$ and $Y$ subject to the single constraint (4).[9] Note that the effective budget restricts the range of values of $L$ in autarkic equilibrium (if $L$ is too close to either 0 or 1, the aggregate production possibility and consumption time loci cannot intersect). This is graphically illustrated in Figure 1 where it is assumed without loss that $a_Y/a_X > b_Y/b_X$. This assumption means that the relative (direct to indirect) price of good $Y$ is greater than that of good $X$. The classic example of cloth (good $X$) and wine (good $Y$) describes this scenario well although cloth is not truly perishable as required in this model. The effective budget is represented by the line $BB'$. The minimum value of $L$ below which the aggregate production possibility and consumption time loci cannot intersect can be derived when the production possibility line $BP'$ and the consumption time line $BT'$ intersect at $B$. This value is given by

$$L_{Min} = a_X/(a_X+b_X) \tag{5}$$

Similarly, the maximum value of $L$ above which the aggregate production possibility and consumption time loci cannot intersect is when the production possibility line $PB'$ and the consumption time line $TB'$ intersect at $B'$. It is given by

$$L_{Max} = a_Y/(a_Y+b_Y) \tag{5'}$$

As $L$ increases from $L_{Min}$ to $L_{Max}$, the aggregate production possibility locus shifts parallel up from $BP'$ to $PB'$ whereas the aggregate consumption time locus shifts

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[9] Note that the values of $X^o$ and $Y^o$, derived from this constrained maximization problem, together with information about $a_X$ and $a_Y$ (or $b_X$ and $b_Y$) will uniquely determine $L^o$. Graphically speaking, the autarkic equilibrium point and the slope $a_X/a_Y$ (or $b_X/b_Y$) uniquely determine the autarkic equilibrium production possibility (consumption time) locus.

parallel down from *BT'* to *TB'*, and the intersections of the two loci trace out the effective budget line *B'B*.

Let us define $p^o$ as the autarkic equilibrium marginal rate of substitution of *Y* for *X*, i.e., $p^o \equiv U_Y(X^o, Y^o)/U_X(X^o, Y^o) = (a_Y+b_Y)/(a_X+b_X)$. It holds true that either

$$a_Y/a_X > p^o > b_Y/b_X \text{ or } b_Y/b_X > p^o > a_Y/a_X. \tag{6}$$

The findings so far can be summarized in

**Proposition 2**: Maximizing social utility by the choice of *L*, *X* and *Y* subject to (1) and (2) is equivalent to maximizing social utility by the choice of *X* and *Y* subject to (4). Further, the autarkic equilibrium $L^o$ is bounded by $a_X/(a_X+b_X) < L^o < a_Y/(a_Y+b_Y)$ if $a_Y/a_X > b_Y/b_X$ (or by $a_X/(a_X+b_X) > L^o > a_Y/(a_Y+b_Y)$ if $a_Y/a_X < b_Y/b_X$) and the autarkic equilibrium price ratio $p^o$ lies strictly between the production and consumption time opportunity costs.

The economy's autarkic equilibrium is illustrated in Figure 2 for the situation $a_Y/a_X > b_Y/b_X$. In this graph, the autarkic equilibrium aggregate production possibility and consumption time loci and the effective budget are depicted by $P^o P^{'o}$, $T^o T^{'o}$ and *BB'*, respectively. At the autarkic equilibrium point *K*, the social indifference curve is tangential to *BB'* but neither to $P^o P^{'o}$ nor to $T^o T^{'o}$, as dictated by (3).

Before moving on to consider the effects of trade, let us briefly examine some simple comparative static results arising from exogenous improvements in production or consumption technologies.[10] Broadly speaking, these improvements can take place in the production (consumption) of either or both goods at the same rate or at

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[10] It is assumed that the saving in the consumption time of a commodity does not alter the utility derived from one unit of that commodity.

different rates. Improvements may happen in either production or consumption exclusively, or in both simultaneously.

From equation (4), it is clear that a reduction in $a_X$ or $a_Y$ or both will alter $p^o$ and the effective budget line accordingly.[11] A similar statement can be made regarding a reduction in $b_X$ or $b_Y$ or both. The change in the autarkic equilibrium consumption bundle $(X^o, Y^o)$ in response to a change in $p^o$ is well described by standard consumer theory and there is no real gain in reproducing these results here. It is therefore sufficient to focus our attention on the range of feasible $L$ and $L^o$ itself.

We note that the effects of technical progress in production of one good are similar to those of trade. These effects will be considered in greater detail in the next section. For the time being, let us focus on the effects of improvements in the consumption technology. For concreteness, let us continue to assume that $a_Y/a_X > b_Y/b_X$. Without loss, consider an improvement in the consumption of good $X$ (i.e., a small reduction in $b_X$) so that the above inequality still holds.[12] From equation (5), it immediately follows that the lower bound of the feasible labour allocation range will become larger. Similarly, if $b_Y$ becomes smaller then the upper bound of the feasible labour allocation range will become larger.

To see how $L^o$ responds to a small reduction in $b_X$, we note that there are potentially two sources of gain from such an improvement:

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[11] In response to a reduction in $a_X$ ($a_Y$) only, the effective budget line will swing in an anticlockwise (clockwise) direction from $B'$ ($B$). Following a simultaneous reduction in both $a_X$ and $a_Y$, the effective budget line will shift upward. In this situation, its slope will be the same, become flatter or become steeper according to whether $a_X$ and $a_Y$ reduce at the same rate, $a_X$ reduces at a higher rate, or $a_Y$ reduces at a higher rate, respectively.

[12] It does not matter if $b_X$ is reduced sufficiently to reverse the direction of the inequality.

- a consumption gain (allowing the consumption bundle to change but holding $L^o$ constant); and

- a time allocation gain (allowing $L^o$ to vary).

This is graphically illustrated in Figures 3-a and 3-b where the dotted lines $BB'$, $P^oP''^o$ and $T^oT''^o$ stand for the pre-improvement effective budget line, equilibrium production locus and equilibrium consumption time locus, respectively. The line $T^1T''^o$ represents the post-improvement consumption time locus holding labour time devoted to production constant at $L^o$. The points $M$ and $N$ are the intersections between $T^1T''^o$ and $P^oP''^o$, and between the pre-improvement autarkic equilibrium social indifference curve and $P^oP''^o$, respectively. The line $B^1B'$ passing through $M$ is thus the post-improvement effective budget line. There are three possibilities:

- $M$ coincides with $N$, i.e., the consumption bundle at $K$ is equally preferred to that at $M$. In this situation, the consumption gain is zero and it is possible to increase welfare by consuming at a point lying to the left of $M$ along $B^1B'$, i.e., by increasing the amount of labour time devoted to production.

- $M$ lies to the right of $N$, i.e., the consumption bundle at $K$ is preferred to that at $M$. In this situation, as shown in Figure 3-a, the consumption gain is negative but it is possible to increase welfare above the pre-improvement level by devoting more labour time to production.

- $M$ lies to the left of $N$, i.e., the consumption bundle at $M$ is preferred to that at $K$. In this situation, as shown in Figure 3-b, the consumption gain is positive. There are now three sub-cases:

(i) *MRS* ($\equiv U_Y/U_X$) at *M* is by mere chance equal to $(a_Y+b_Y)/(a_X+b_X^1)$ where

$b_X^1$ is the new value of $b_X$, i.e., the social indifference curve passing through

*M* is tangential to $B^1B'$ at *M*. In this sub-case, *M* is also the post-improvement

equilibrium, $L^o$ remains unchanged and the time allocation gain is zero.

(ii) *MRS* at *M* is greater than $(a_Y+b_Y)/(a_X+b_X^1)$, i.e., the social indifference

curve passing through *M* intersects $B^1B'$ to the left of *M*. In this sub-case, a

positive time allocation gain can be obtained by raising the value of $L^o$.

(iii) *MRS* at *M* is less than $(a_Y+b_Y)/(a_X+b_X^1)$, i.e., the social indifference curve

passing through *M* intersects $B^1B'$ to the right of *M*. In this sub-case, a

positive time allocation gain can be realized by reducing the value of $L^o$.

We are now ready to state

**Proposition 3**: In response to an exogenous improvement in the consumption

technology of one good, the range of the feasible amount of labour devoted to

production expands. When the total gain from such an improvement is strictly

positive, the consumption gain (holding time allocation constant) can be positive,

zero or negative and the time allocation gain can be zero or positive. If the

consumption gain is positive and *MRS* at *M* is greater than (or equal to)

$(a_Y+b_Y)/(a_X+b_X^1)$, it is welfare increasing to reduce (maintain) the pre-improvement

amount of labour devoted to production. In all other cases, a strictly positive time

allocation gain can be realized by devoting more labour to production..

The interpretation of the second part of Proposition 3 is straightforward enough.

An improvement in consumption technology will enable the economy to consume a

'greater' bundle of goods so that it is welfare increasing to allocate more labour time to production and less labour time to consumption.

## 3. A small open Ricardian economy

The country is now open to free, costless and balanced trade in commodities. The home country is small in the sense that it takes the world terms of trade $p^e$ (price of $Y$ in terms of $X$) as given. In the context of trade, production and consumption must be distinguished and this distinction will be made whenever necessary. In this section, unless otherwise stated, $X$ and $Y$ will denote the home country's consumption of the two goods.

To examine the impact of trade on the home country's production and consumption, it is sufficient to see how its effective budget line changes in response to trade. Without loss of generality, let us assume that $a_Y/a_X > b_Y/b_X$. There are three cases as anticipated by the traditional principle of comparative advantage:

Case (a): $p^e > a_Y/a_X$

The home country has a comparative advantage in the production of good $Y$. At any feasible $L$, by completely specializing in the production of $Y$ and trading at the world terms of trade $p^e$, the home country can afford (in the income sense) any consumption bundle $(X, Y)$ where $Y = L/a_Y - E$, $X = p^e E$ and $E$ ($\geq 0$) stands for its exports of good $Y$. Eliminating $E$, the equation of this particular production-cum-trade possibility locus[13] is

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[13] This describes the set of commodity bundles that can be obtained by production (complete specialization) and trade, given $p^e$ and $L$.

$(a_Y/p^e)X + a_Y Y = L.$ (7)

Consider an intersection of the home country's production-cum-trade possibility and consumption time loci at any feasible $L$. The locus of all such intersections is the home country's post-trade effective budget, the equation of which can be obtained by adding (7) and (2):

$(a_Y/p^e + b_X)X + (a_Y + b_Y)Y = 1$ (7')

The post-trade effective budget line is equivalent to the consumption possibility locus in the standard Ricardian model. Graphically speaking, the home country's effective budget line swings from $B'$ in an anticlockwise direction to $B'D$ (a dark line in Figure 4). Note that, as $p^e$ approaches $\infty$ from $a_Y/a_X$, the magnitude of the slope of $B'D$ ($\equiv (a_Y/p^e + b_X)/(a_Y + b_Y)$) decreases steadily and is bounded above by $(a_X + b_X)/(a_Y + b_Y)$ and strictly below by $b_X/(a_Y + b_Y)$.

Case (b): $p^e = a_Y/a_X$:

The home country's effective budget remains unchanged and trade is annihilated.

Case (c): $p^e < a_Y/a_X$

A symmetrical argument to case (a) applies. For any feasible $L$, the home country can completely specialize in the production of $X$, trade at $p^e$ and afford (in the income sense) any consumption bundle along its production-cum-trade possibility locus:

$a_X X + a_X p^e Y = L.$ (8)

The equation for the home country's post-trade effective budget line thus becomes

$(a_X + b_X)X + (a_X p^e + b_Y)Y = 1.$ (8')

As depicted in Figure 4, the home country's effective budget line swings from $B$ in a clockwise direction to $BC$. Note that, as $p^e$ approaches 0 from $a_Y/a_X$, the

magnitude of the slope of $BC$ ($\equiv (a_X+b_X)/(a_Xp^e+b_Y)$) increases steadily and is bounded below by $(a_X+b_X)/(a_Y+b_Y)$ and strictly above by $(a_X+b_X)/b_Y$.

The introduction of trade disturbs the range of the feasible post-trade equilibrium $L^e$. Continuing to assume that $a_Y/a_X > b_Y/b_X$ and following the same analysis as in the previous section, it can be shown that

- if $p^e > a_Y/a_X$ then $a_Y/(a_Y+p^eb_X) < L^e < a_Y/(a_Y+b_Y)$;

- if $a_Y/a_X > p^e$ and $p^e \neq b_Y/b_X$ then $a_X/(a_X+b_X) < L^e < a_Xp^e/(b_Y+a_Xp^e)$; and

- if $p^e = b_Y/b_X$ then $L^e = a_X/(a_X+b_X)$, i.e., the range of the feasible $L^e$ collapses into a single point.

Bearing in mind that $a_X/(a_X+b_X) < L^o < a_Y/(a_Y+b_Y)$, it is easy to see that:

- if $p^e > a_Y/a_X$ then $a_Y/(a_Y+p^eb_X) < a_X/(a_X+b_X)$, i.e., the lower bound of the range of the feasible amount of labour devoted to production becomes smaller; and

- if $a_Y/a_X > p^e$ and $p^e \neq b_Y/b_X$ then $a_Xp^e/(b_Y+a_Xp^e) < a_Y/(b_Y+a_Y)$, i.e., the upper bound of the range of the feasible amount of labour devoted to production becomes smaller.

Let $MRS^e \equiv U_Y(X^e, Y^e)/U_X(X^e, Y^e)$ be the post-trade equilibrium marginal rate of substitution of $Y$ for $X$ where $X^e$ and $Y^e$ stand for the post-equilibrium consumption of commodities $X$ and $Y$, respectively. From equations (7') and (8') it can be shown that

$$MRS^e = \begin{cases} p^e(a_Y+b_Y)/(a_Y+b_Xp^e) < p^e & p^e > a_Y/a_X \\ p^e(a_X+b_Y/p^e)/(a_X+b_X) < p^e & a_Y/a_X > p^e > b_Y/b_X \\ p^e(a_X+b_Y/p^e)/(a_X+b_X) = p^e & p^e = b_Y/b_X \\ p^e(a_X+b_Y/p^e)/(a_X+b_X) > p^e & p^e < b_Y/b_X \end{cases} \tag{9}$$

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We are now ready to state the small-open-economy counterpart of Proposition 2.

**Proposition 4**: The patterns of (complete) specialization and trade in this extended model are dictated by the conventional Ricardian principle of comparative advantage. However, the range of the feasible amount of labour devoted to production contracts in the post-trade situation. In particular, for the situation $a_Y/a_X > b_Y/b_X$, this range collapses to a single point $L^e = a_X/(a_X+b_X)$ when $p^e = b_Y/b_X$. Further, the post-trade equilibrium *MRS* is smaller than, equal to or greater than $p^e$ according to whether $p^e$ is greater than, equal to or less than $b_Y/b_X$, respectively.

Let us now explore the sources of gains from trade in this extended model. The traditional Ricardian approach distinguishes between consumption gain (which is attributable to consuming at prices different from autarkic prices, holding production unchanged) and production gain (which is attributable to specialization according to the principle of comparative advantage). In the extended model of this section, there are potentially three sources of gains from trade:

- an exchange gain (holding both time allocation and production mix constant);
- a specialization gain (holding time allocation constant but allowing production mix to vary); and
- an allocation gain (allowing the time allocation to vary).

Each of the above will be considered in turn.

Without loss, we again assume that $a_Y/a_X > b_Y/b_X$. The specialization gain is illustrated in Figure 5. In this graph, $K$ is the autarkic equilibrium, $P^oK_1$ is the production-cum-trade possibility locus corresponding to the world terms of trade $p_1^e$

($< a_Y/a_X$) at the autarkic $L^o$ and $P''^o K_2$ is the production-cum-trade possibility locus corresponding to the world terms of trade $p_2^e$ ($> a_Y/a_X$) also at the autarkic $L^o$. It is clear that:

- Due to the consumption time constraint, the exchange gain (which is equivalent to the consumption gain in the traditional model) vanishes in this extended model.

- Let $Z$ be the intersection of the autarkic equilibrium indifference curve and the autarkic consumption time constraint $T^o T''^o$. If the slope of $P''^o Z$ is nonnegative (as in Figure 5), then the specialization gain, represented by the movement from $K$ to $K_2$, is always positive for any $p^e > a_Y/a_X$. If the slope of $P''^o Z$ is negative then the specialization gain is positive only for $p^e < \bar{p}$ where $\bar{p}$ is the magnitude of the reciprocal of the slope of $P''^o Z$. If $p^e < a_Y/a_X$ then the specialization gain, represented by the movement from $K$ to $K_1$, is positive only for $p^e > \underline{p}$ where $\underline{p}$ is the magnitude of the reciprocal of the slope of $P^o Z$. In general, the specialization gain in the situation $a_Y/a_X > b_Y/b_X$ is positive for $\underline{p} < p^e < \bar{p}$ where $\bar{p}$ can be infinity. Outside this range, the specialization gain can be zero or negative.[14] Symmetrical results apply in the situation $a_Y/a_X < b_Y/b_X$, i.e., the specialization gain is positive for $\underset{\sim}{p} < p^e < \tilde{p}$ where $\underset{\sim}{p}$ can be zero.

The time allocation gain is depicted in Figure 6 assuming that $a_Y/a_X > b_Y/b_X$ and $p^e > a_Y/a_X$. In this graph, the dotted line $BB'$ is the home country's autarkic effective

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[14] Note that the specialization gain arising from trade is similar to the consumption gain arising from the improvement in the consumption technology of one good.

budget line and $K$ is its autarkic equilibrium. The world terms of trade is given by the magnitude of the reciprocal of the slope of $P''^oK_2$. $B'D$ is the home country's post-trade effective budget line[15] and $E$ is its post-trade equilibrium. The home country's gains from trade can be decomposed into two components:

- a specialization gain, represented by the movement from $K$ to $K_2$; and

- an time allocation gain, represented by the movement from $K_2$ to $E$.

More importantly, since the home country's post-trade equilibrium point $E$ lies entirely to the right of the autarkic consumption time constraint $T^oT''^o$, it implies that the post-trade $L^e$ is necessarily smaller than autarkic $L^o$. Similar results can be easily obtained for the case $p^e < a_Y/a_X$. We can summarize these findings in

**Proposition 5**: For $p^e \neq a_Y/a_X$, trade can take place and it is strictly gainful to a small country. While the exchange gain is annihilated in the extended model, the specialization gain is positive only for a well defined range of $p^e$. Outside this range, the specialization gain can be zero or negative. In any case, a small trading nation can further increase its post-trade welfare by relocating labour time from production to consumption (i.e., $L^e < L^o$).

The intuition of the last part of Proposition 5 is clear. Under trade, the home country can always consume, if it so wishes, a larger bundle of goods than the autarkic equilibrium bundle $K$.[16] It is therefore welfare improving to spend more time on consumption.

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[15] From equation (8), it is clear that, in this situation, the slope of $P''^oK_2$ ($1/p^e$) is flatter than that of $B'D$ (($a_Y/p^e+b_X)/(a_Y+b_Y)$).
[16] Any bundle lying on $B'D$ to the north east of $K$ is both greater than the autarkic equilibrium bundle $K$ and affordable (both in the income and time sense).

Figure 7 illustrates the home economy's post-trade equilibrium production and consumption assuming that $a_Y/a_X > b_Y/b_X$ and $p^e > a_Y/a_X$. In this graph, the lines $P^e P^{\prime\prime e}$ and $T^e T^{\prime\prime e}$ are the home country's post-trade equilibrium aggregate production possibility and consumption time loci, respectively. The world terms of trade is given by the magnitude of the reciprocal of the slope of $P^{\prime\prime e} E$ (the same as that of $P^{\prime\prime o} K_2$). The home country's pattern of trade is given by the triangle $P^{\prime\prime e} E Y^e$ where $P^{\prime\prime e} Y^e$ represents the exports of good $Y$ and $E Y^e$ the imports of good $X$. Post-trade equilibrium consumption is then the bundle $(X^e, Y^e)$. As anticipated by standard consumer theory, $X^e > X^o$ (assuming that $X$ is not a Giffen good).

The effects of a small change in the world terms of trade on post-trade labour allocation, production, volume of trade and consumption can also be deduced in a straightforward manner. In Figure 7, for example, a small increase (decrease) in $p^e$ (which maintains $p^e > a_Y/a_X$) will pivot the post-trade effective budget line from $B'$ in an anticlockwise (clockwise) direction. As a result, $L^e$ will become smaller (larger), post-trade production of $Y$ smaller (larger), exports of $Y$ larger (smaller), $X^e$ larger (smaller) and $Y^e$ smaller (larger).

## 4. A Ricardian world economy

We are now ready to examine a Ricardian world economy consisting of two large economies: the home and foreign countries. For ease of notation, let us denote the variables associated with the foreign country by an asterisk. Given any particular world terms of trade, we can determine the profit-maximizing pair of the home

country's outputs, uniquely except when the hypothetical world terms of trade are equal to $a_Y/a_X$, and we can determine the utility-maximizing home country's consumption pair. This information generates the home country's offer curve. The foreign country's offer curve can be similarly obtained. Assuming that $a_Y/a_X <$ $a_Y^*/a_X^*$, the two offer curves are graphed together in Figure 8 where $W$ represents the world trading equilibrium and $p^W$ the equilibrium world terms of trade. All standard results thus emerge from this construction.[17]

**Proposition 6**: If $a_Y/a_X$ and $a_Y^*/a_X^*$ differ then (i) a non-trivial world equilibrium exists, (ii) the equilibrium terms of trade is strictly bounded by $a_Y/a_X$ and $a_Y^*/a_X^*$, and (iii) both countries can benefit from engaging in trade.

Very recently Kemp (2008) has shown that the Kemp–Wan gains-from-trade proposition remains valid even under very general Gossenian and Walrasian assumptions. Proposition 6 in this paper illustrates his result for the case of a Ricardian world economy.

Note that we have so far restricted international trade to trade in commodities. This is equivalent to technical progress in the productive technology of one commodity. But international trade could also encompass trade in ideas. Residents in one country may learn how to consume a particular commodity more quickly from residents of another country. This results in improvements in consumption technologies. But trade in ideas is conceptually more difficult to formulate and analyze (than trade in commodities), and will not be attempted here.

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[17] Kemp and Okawa (2006) have shown that the principle of comparative advantage ceases to be completely valid if one or both countries produce and consume only one commodity under autarky.

## 5. Summary conclusion

In this present note we explore the implications of incorporating a Gossenian–Beckerian consumption time constraint into a simple Ricardian model of trade. Not surprisingly, the autarkic equilibrium full price ratio is shown to lie strictly between the production and consumption opportunity costs. In response to an exogenous improvement in the consumption technology of one good, the range of the amount of labour that might feasibly be allocated to production expands and more labour time is devoted to production.

It is also shown that the range of the feasible allocation of labour contracts as a result of trade. Further, while the traditional principle of comparative advantage remains intact, the sources of the gains from trade are different. First, the consumption gain from trade is annihilated because of the time constraint on consumption. Secondly, the production gain from trade is generally speaking positive only for a well defined range of world terms of trade. Finally and perhaps most interestingly, a Ricardian open economy can further enjoy a time allocation gain by relocating its labour resources from production to consumption.

On the consumption side, this paper assumes linear time constraint. This is not crucial as similar results can be derived under the assumption of concave or convex consumption time constraints. On the production side, this paper assumes constant return to scale with one factor of production. As suggested in Kemp (2008), the normative results obtained in this paper carry over to the Heckscher–Ohlin–

Samuelson model. However, it would be difficult to derive results as clear cut as those of this paper.

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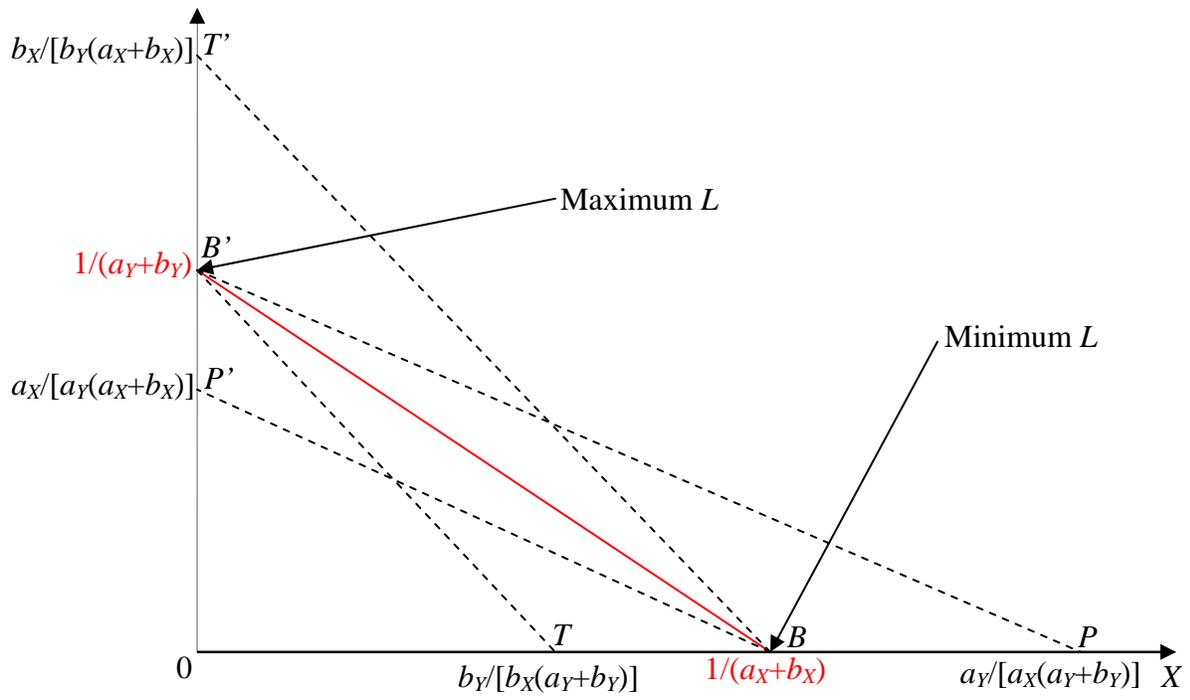Steedman, I., 2001. Consumption Takes Time, Routledge, London.

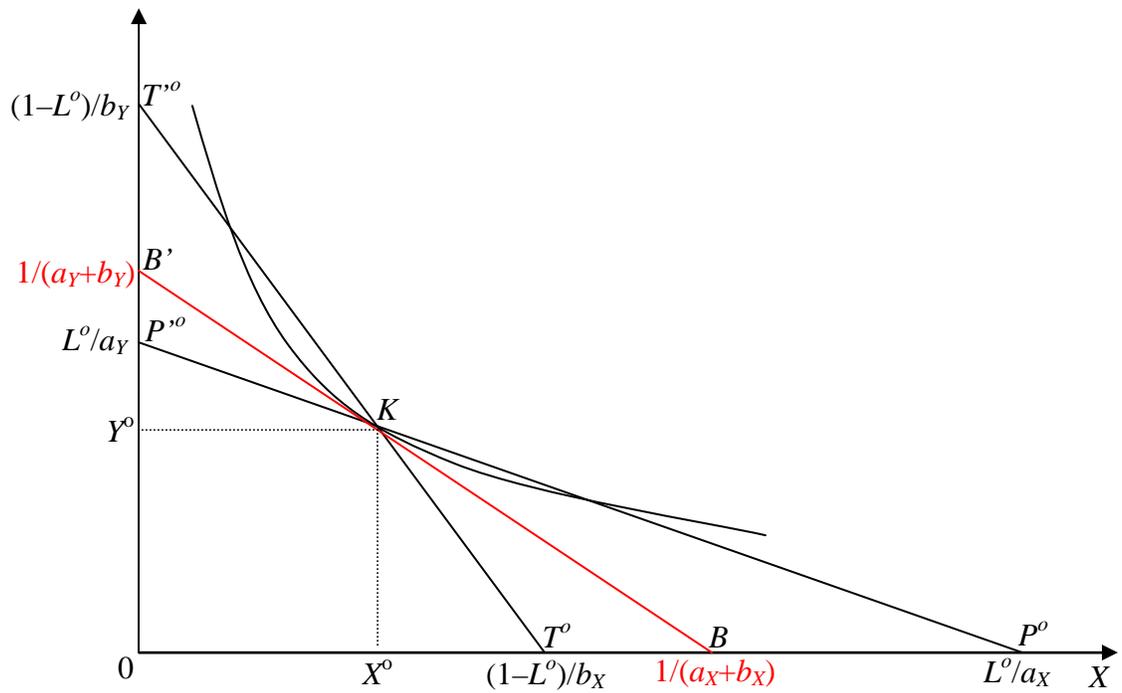Figure 1: Effective budget line and range of feasible labour ($a_Y/a_X > b_Y/b_X$)



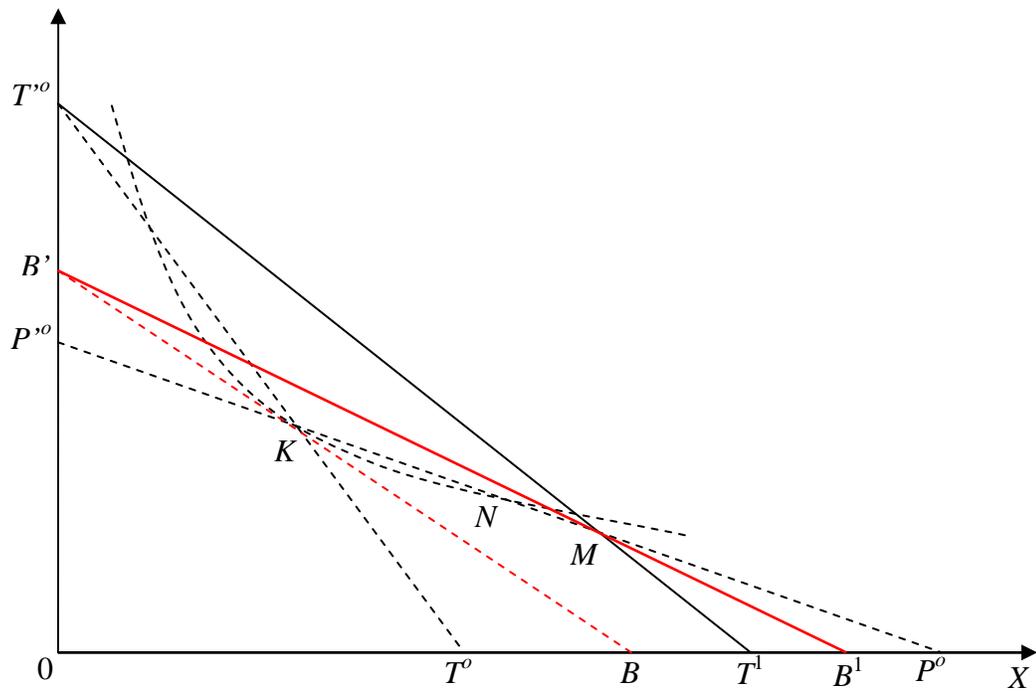Figure 2: Autarkic equilibrium ($a_Y/a_X > b_Y/b_X$)

Figure 3-a: Improvement in the consumption technology of commodity $X$
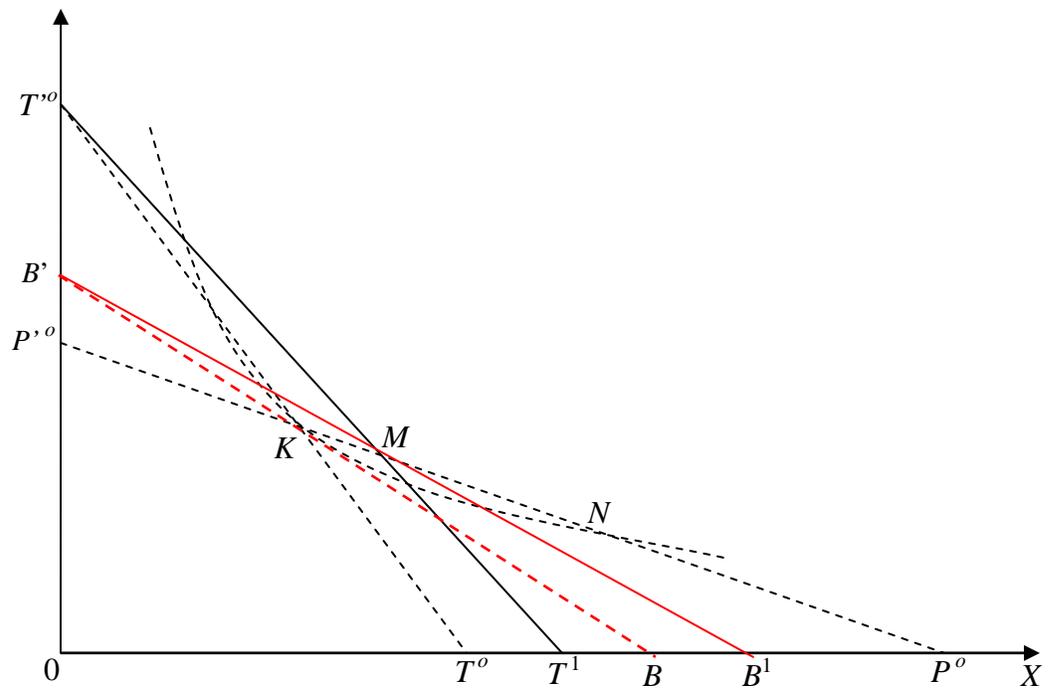with a negative consumption gain ($a_Y/a_X > b_Y/b_X$)



Figure 3-b: Improvement in the consumption technology of commodity $X$
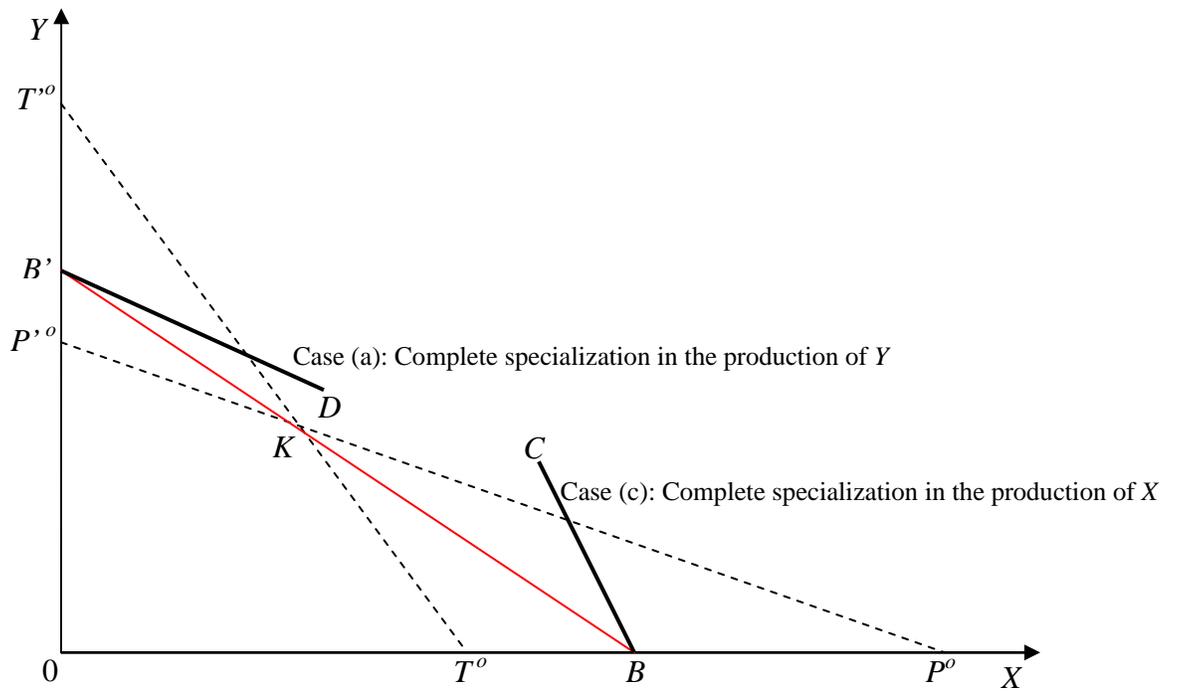with a positive consumption gain ($a_Y/a_X > b_Y/b_X$)

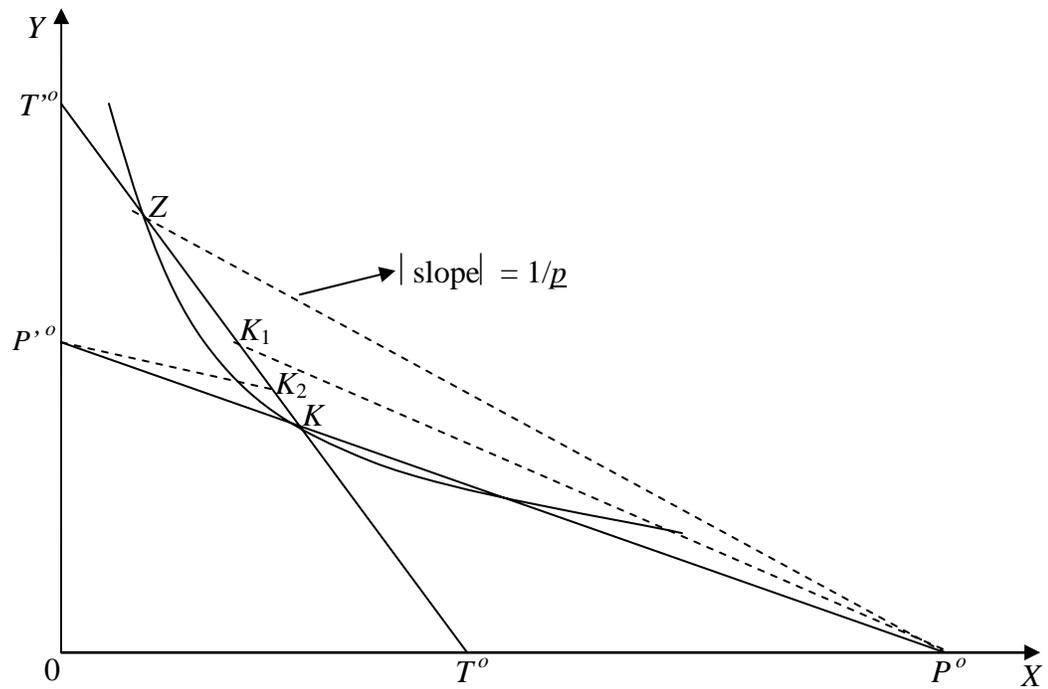Figure 4: Post-trade production specialization ($a_Y/a_X > b_Y/b_X$)



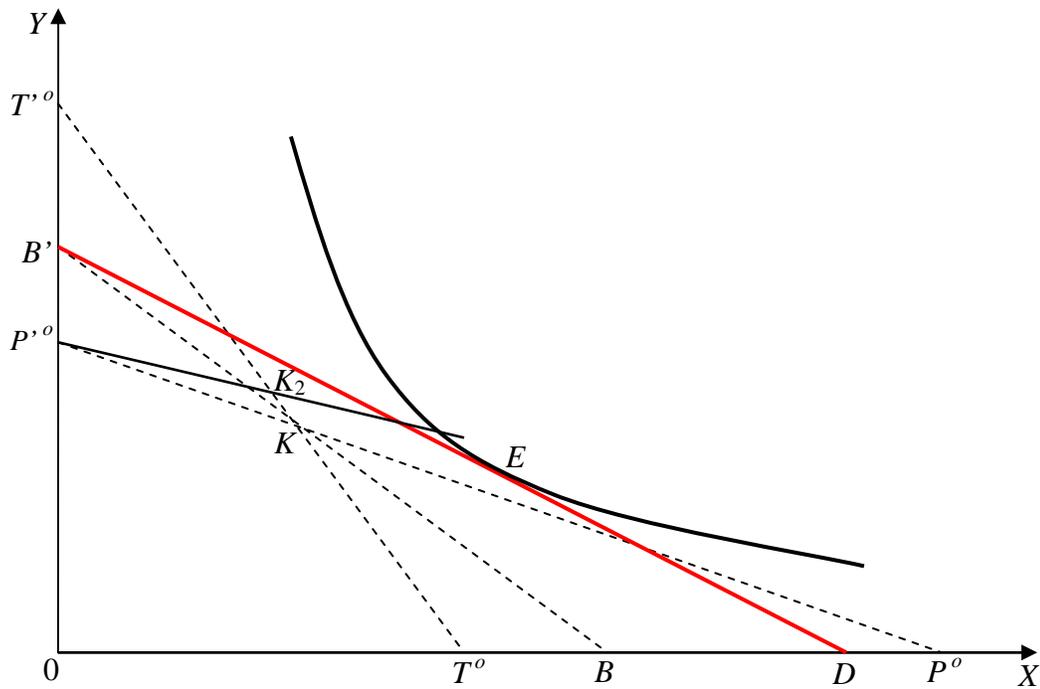Figure 5: Specialization gain from trade ($a_Y/a_X > b_Y/b_X$)

Figure 6: Gains from trade ($a_Y/a_X > b_Y/b_X$ and $p^e > a_Y/a_X$)
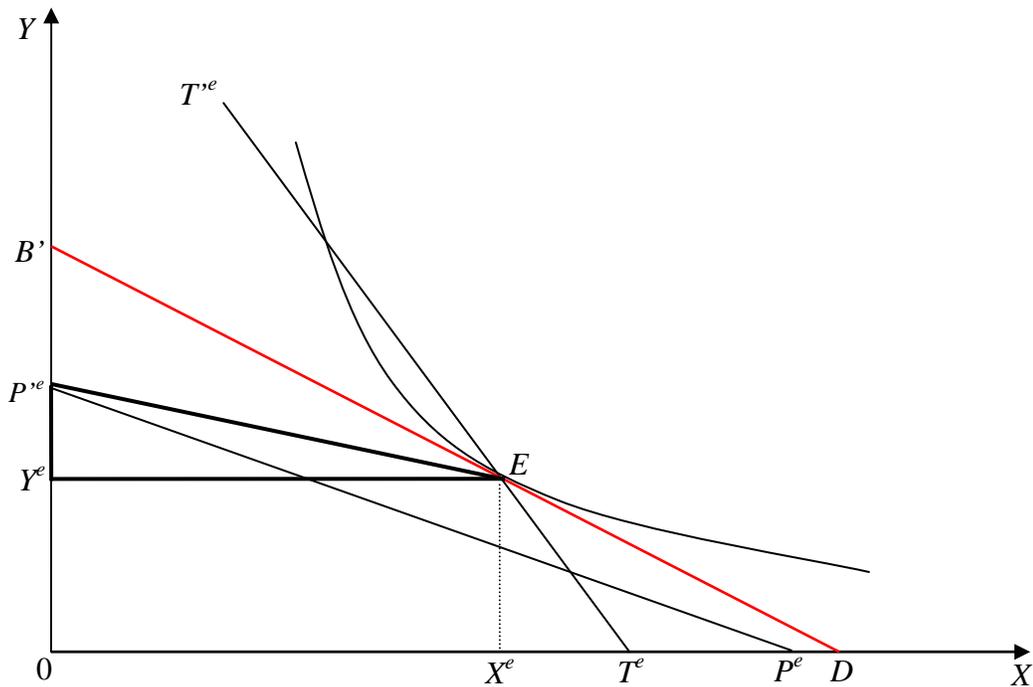


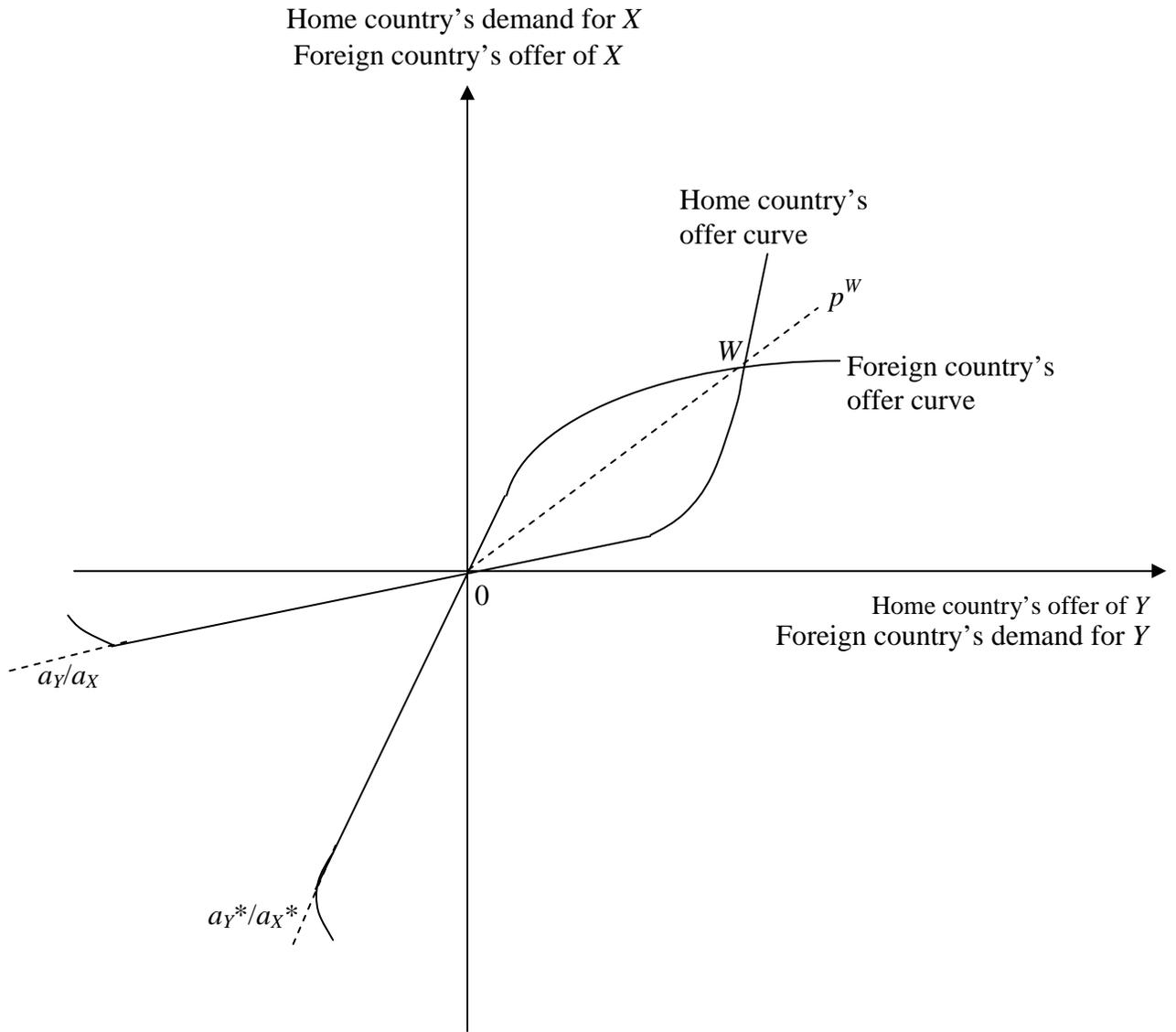Figure 7: Pattern of trade ($a_Y/a_X > b_Y/b_X$ and $p^e > a_Y/a_X$)

Home country's demand for $X$
Foreign country's offer of $X$

Home country's
offer curve

$p^W$

$W$

Foreign country's
offer curve

0

Home country's offer of $Y$
Foreign country's demand for $Y$

$a_Y/a_X$

$a_Y*/a_X*$

Figure 8: World trading equilibrium ($a_Y/a_X < a_Y*/a_X*$)