Partial privatization as a source of trade gains

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Abstract

A model of mixed oligopoly is constructed in which a Home public firm competes with Home private firms and Foreign private firms in the domestic market to consider how *partial* privatization affects trade gains. In the short-run with the fixed number of firms, a progress in privatization is more likely to make trade gainful if the public firm is insufficiently privatized. In contrast, gains prove positive irrespective of the degree of privatization in the long-run with free entry, namely, the presence of public firms itself serves as a source of gains. It is also shown how these results are modified when the government optimally privatizes the public firm to maximize welfare.

textitKeywords: gains from trade, partial privatization, mixed oligopoly, free entry.

JEL classification: F12, L32, L33.

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1 Introduction

The purpose of this paper is to construct a model of mixed oligopoly to study how partial privatization affects gains from trade. Since the latter half of 1990s, there is a growing body of literature on mixed oligopoly with foreign competitors, which is divided into two strands.

The first generation evaluates the effects of privatization by comparing a fully privatized equilibrium with a fully nationalized equilibrium. Employing an extended model of De Fraga and Delbono (1989), Fjell and Pal (1996) provide a model in which a Home public firm competes with Home private firms and Foreign private firms to examine the effect of trade captured by an exogenous increase in the number of Foreign firms. Incorporating trade policies, Pal and White (1998) study how privatization affects welfare when the Home government optimally chooses a tariff or production subsidy. Barcena-Ruiz and Garzon (2005), on the other hand, consider a mixed oligopoly in an integrated world market to explore under what condition each country's government is motivated to privatize. In other words, they are interested in strategic aspects of two countries' government over privatization.

While these works focus on full privatization, recent studies comprising Chang (2005, 2007), Chao and Yu (2006), and Long and Staehler (2007) allow for *partial* privatization based on Matsumura's (1998) formulation. According to it, the public firm maximizes a weighted average of profit and welfare. Then, privatization is captured by an increase in the weight put on profit. The public firm is fully privatized when the weight on profit is one, i.e., its objective function is profit. Chang (2005, 2007) and Chao and Yu (2006) analyze how the conventional wisdom on the optimum tariff is influenced by partial privatization. Long and Staehler (2007) provide a variety of models of mixed oligopoly to reconsider the optimal tariff and production subsidy in the presence of partial privatization.

While all of the above literature is interested in trade policy issues, there is no study that relates partial privatization to gains from trade, which is fulfilled in this paper. Our model shares the existing ones in many respects, but it is helpful to comment what differentiates this paper from the predecessors. First, we allow for partial privatization

unlike Fjell and Pal (1996). Second, we consider a market in which a Home public firm competes with an arbitrary number of Home private firms and Foreign private firms, which sharply contrasts with Chang (2005, 2007) and Chao and Yu (2006) who ignore the presence of Home private firms. Third, the existing literature has neglected the role of free entry among private firms, which is a main concern in Matsumura and Kanda (2005) who assume a closed economy. By contrast, more attention is paid to free entry in this paper.

We prove two main results. First, the Home country gains from trade if the number of Foreign firms is sufficiently larger than that of Home firms. This result holds under private oligopoly as well, but it is conditional whether privatization favorably affects gainfulness of trade. If the public firm is initially nationalized enough, privatization raises the possibility of gains from trade. Second, in the long-run with free entry, positive gains from trade are ensured regardless of the level of privatization. Both of these results could give an affirmative evaluation to privatization which international institutions, e.g., the WTO and IMF, have been promoting.

The paper is structured as follows. Section 2 sets up a basic model and characterizes a short-run equilibrium with entry prohibited. Section 3 derives the condition for positive gains from trade and examines how it is affected by privatization. Allowing for free entry, Section 4 further considers welfare effects of trade liberalization. Section 6 endogenizes the choice of the degree of privatization and investigates gains from trade under optimal privatization. Finally, Section 5 concludes the paper.

2 A model

Consider a country, say Home, which produces two goods from one primary factor, labor. Without loss of generality, one unit of labor produces one unit of Good 2 (numeraire) so that the wage rate is fixed to unity. On the other hand, the market if Good 1 is oligopolized by one public firm (indexed 0), $n \geq 0$ Home private firms, and $n^* \geq 0$ Foreign private firms. An asterisk (*) is attached to Foreign variables. All the firms share the identical cost which involves a constant marginal cost $c \geq 0$ and a fixed cost $f \geq 0$. In our context, autarky is captured by $n^* = 0$, namely no Foreign firm.

Assume a representative consumer whose utility function is

$$u = aC_1 - \frac{C_1^2}{2} + C_2, \quad a > c, \tag{1}$$

where u is utility and $C_i,\ i = 1, 2$ the consumption of each good. Utility maximization subject to the budget constraint yields a linear demand function: $C_1 = a - p$, where p denotes the price of Good 1 measured by Good 2. Then, the inverse demand function is

$$p = a - X, \quad X \equiv x_o + \sum x_i + \sum x_i^*,$$

where x_0 is the public firm's output and x_i (resp. x_i^*) the output of a representative private firm in Home (resp. Foreign). Under these specifications, each firm's profit is respectively defined by

$$\pi_0 = (a - c - X)x_0 - f \tag{2}$$

$$\pi_i = (a - c - X)x_i - f \tag{3}$$

$$\pi_i^* = (a - c - X)x_i^* - f. \tag{4}$$

Given the utility function (1), the Home welfare is measured by

$$U = \frac{X^2}{2} + \pi_0 + \sum \pi_i, \tag{5}$$

where U is social welfare and $X^2/2$ in the right-hand side consumer surplus. As in the existing literature such as Matsumura and Kanda (2005), Chang (2005), and Chao and Yu (2006), the public firm seeks to maximize

$$\begin{aligned} & \theta\pi_0 + (1 - \theta)U \\ = \ & \pi_0 + (1 - \theta)\left(\frac{X^2}{2} + \sum \pi_i\right), \quad \theta \in [0, 1], \end{aligned} \tag{6}$$

where θ is a parameter which measures the degree of privatization. We will occasionally say that the public firm is fully privatized (resp. nationalized) under $\theta = 1$ (resp. $\theta = 0$). On the other hand, each private firm chooses output to maximize profits.

Let us now characterize a short-run equilibrium where the number of firms is fixed. Supposing that each firm plays a Cournot-Nash game, the system of first-order condition

for objective maximization is

$$\begin{bmatrix} \theta+1 & n & \theta n^* \\ 1 & n+1 & n^* \\ 1 & n & n^*+1 \end{bmatrix} \begin{bmatrix} x_0 \\ x \\ x^* \end{bmatrix} = \begin{bmatrix} a-c \\ a-c \\ a-c \end{bmatrix},$$

where use is made of the symmetry assumption such that $x_i = x_j = x$ and $x_i^* = x_j^* = x^*$. Solving this system for respective outputs yields

$$x_0 = \frac{[n^*(1-\theta)+1](a-c)}{\theta(n+1)+n^*+1} \tag{7}$$

$$x = x^* = \frac{\theta(a-c)}{\theta(n+1)+n^*+1}. \tag{8}$$

Summing up (7) and (8), the total supply is

$$X = x_0 + nx + n^*x^* = \frac{(\theta n + n^* + 1)(a-c)}{\theta(n+1)+n^*+1}. \tag{9}$$

Substituting (7)-(9) into (5), the Home welfare is computed as follows.

$$\begin{aligned} U^T &= \frac{X^2}{2} + \pi_0 + \pi \\ &= \frac{X^2}{2} + (a-c-X)(x_0+nx) - (n+1)f \\ &= \frac{1}{2}\left[\frac{a-c}{\theta(n+1)+n^*+1}\right]^2 \Big\{(\theta n + n^* + 1)^2 + 2\theta[\theta n + n^*(1-\theta)+1]\Big\} - (n+1)f, \end{aligned} \tag{10}$$

where the superscript T indicates the free trade equilibrium.

By the assumption that autarky is given by $n^* = 0$, welfare under autarky is

$$U^A = \frac{1}{2}\left[\frac{a-c}{\theta(n+1)+1}\right]^2 (\theta n+1)[\theta(n+2)+1] - (n+1)f, \tag{11}$$

where the superscript A denotes the autarkic equilibrium. This completes describing a basic model and short-run equilibrium. In the subsequent sections, we apply this model to discuss gainfulness of free trade, i.e., the change from $n^* = 0$ to $n^* > 0$.

3 Gains from trade

Having described the short-run equilibrium with no entry, this section examines the condition for positive gains from trade in the presence of a public firm. The first main

result summarizes it.

Proposition 1. *Home gains from trade if and only if*

$$n^* > \frac{2(\theta n+1)[\theta(n+1)+1]}{\theta}. \tag{12}$$

Proof. From (10) and (11), $U^T > U^A$ is equivalent to

$$\frac{[\theta(n+1)+1]^2\left\{(\theta n+n^*+1)^2+2\theta[\theta n+n^*(1-\theta)+1]\right\}}{[\theta(n+1)+n^*+1]^2(\theta n+1)[\theta(n+2)+1]} > 1.$$

In order to derive the condition for establishing this inequality, subtracting the denominator from the numerator and rearranging terms yield

$$\begin{aligned} & [\theta(n+1)+1]^2\left\{(\theta n+n^*+1)^2+2\theta[\theta n+n^*(1-\theta)+1]\right\} \\ & -[\theta(n+1)+n^*+1]^2(\theta n+1)[\theta(n+2)+1] \\ = \;& \theta n^*\left\{\theta n^*-2(\theta n+1)[\theta(n+1)+1]\right\}. \end{aligned}$$

Thus, we have arrived at the condition equivalent to $U^T > U^A$ given in (12). **Q. E. D.**

Having algebraically derived the condition for gains from trade, let us interpret it intuitively. The opening of trade influences welfare through two channels in our model. First, trade accompanied by foreign entry is procompetitive, i.e., it makes the free trade price lower than the autarkic price, which improves welfare by raising consumer surplus. Second, trade induces profit losses for Home firms, which is harmful to welfare. Due to these conflicting effects, it is not trivial that is necessarily gainful. Condition (12) requires the former positive effect to overweigh the latter negative effect.

Then, our next concern is how privatization affects condition (12). To see this, it is useful to consider the following simple example. Let us try $\theta = 1/2$ and $n = 2$. Then, the right-hand side in (12) becomes 20. However, under $\theta = 1$ and $n = 2$, namely, the public firm is fully privatized, the threshold becomes 24. This simple example numerical example suggests that a progress of privatization makes the right-hand side in (12) higher, which implies that positive gains are more unlikely.

The above argument is now more rigorously considered. Differentiate the right-hand side in (12) with respect to θ to get

$$\frac{\partial}{\partial\theta}\left\{\frac{2(\theta n+1)[\theta(n+1)+1]}{\theta}\right\}=\frac{\theta^2 n(n+1)-1}{\theta^2}.$$

Making use of this calculation, we can state:

Corollary 1. *Privatization is more likely to ensure positive gains from trade if the initial degree of privatization satisfies*

$$\theta<\sqrt{\frac{1}{n(n+1)}}. \tag{13}$$

Under (13), an increase in θ, i.e., a progress of privatization, decreases the right-hand side in (12). Therefore, inequality (12) is more likely to hold and it is fair to say that privatization raises the possibility of gainful trade if the public firm is initially nationalized enough.

Roughly speaking, θ is small in developing countries as compared to developed countries. For such countries, privatization is expected to play a stimulating role for them to gain from trade. In this sense, the above corollary could provide a theoretical rationale for why international institutions represented by the IMF and WTO are encouraging developing countries to privatize the public firm.

4 Long-run equilibrium

In the previous sections, attention has been confined to the equilibrium with no entry. Relaxing this assumption, we now turn to the long-run equilibrium in which free entry among private firms is allowed.

Substituting (9) into (3) and (4), all the Home and Foreign private firms achieve the same maximized profit:

$$\begin{aligned}\pi=\pi^* &= (a-c-X)x-f\\ &= \left\{a-c-\frac{(\theta n+n^*+1)(a-c)}{\theta(n+1)+n^*+1}\right\}\frac{\theta(a-c)}{\theta(n+1)+n^*+1}-f\end{aligned}$$

$$= \left[\frac{\theta(a-c)}{\theta(n+1)+n^*+1}\right]^2 - f. \tag{14}$$

Free entry drives (14) to zero, which yields

$$\theta n^T + n^{*T} = \frac{\theta(a-c)}{\sqrt{f}} - \theta - 1, \tag{15}$$

by setting $\pi = 0$. Applying the same procedure to autarky with $n^* = 0$, we see that

$$\theta n^A = \frac{\theta(a-c)}{\sqrt{f}} - \theta - 1. \tag{16}$$

Thus, it follows from (15) and (16) that $\theta n^T + n^{*T} = \theta n^A$, which will play a helpful role in the subsequent argument.

In view of the considerations so far, we can establish:

Proposition 2. *Under free entry among private firms, trade is necessarily gainful unless the public firm is either fully privatized or fully nationalized.*

Proof. Under free entry with $\pi = 0$, the Home welfare consists of consumer surplus and the public firm's profit:

$$\begin{aligned} U^T &= \frac{X^2}{2} + (a-c-X)x_0 - f \\ &= \frac{1}{2}\left[\frac{a-c}{\theta(n+1)+n^*+1}\right]^2 \left\{(\theta n + n^* + 1)^2 + 2\theta[n^*(1-\theta)+1]\right\} - f. \end{aligned} \tag{17}$$

Setting $n^* = 0$ yields the counterpart under autarky:

$$U^A = \frac{1}{2}\left(\frac{a-c}{\theta n + n^* + 1}\right)^2 \left[(\theta n + 1)^2 + 2\theta\right] - f. \tag{18}$$

Relating the auxiliary result that $\theta n^T + n^{*T} = \theta n^A$ to (17) and (18), we have

$$\begin{aligned} U^T &= \frac{1}{2}\left(\frac{a-c}{\theta n^T + n^{*T} + 1 + \theta}\right)^2 \left\{\left(\theta n^T + n^{*T} + 1\right)^2 + 2\theta\left[n^{*T}(1-\theta)+1\right]\right\} - f \\ &= \frac{1}{2}\left(\frac{a-c}{\theta n^A + 1 + \theta}\right)^2 \left\{\left(\theta n^A + 1\right)^2 + 2\theta + 2\theta(1-\theta)n^{*T}\right\} - f \\ &= U^A + \left(\frac{a-c}{\theta n^A + 1 + \theta}\right)^2 \theta(1-\theta)n^{*T} \end{aligned}$$

$$
\begin{aligned}
&= U^A + \left[\frac{a-c}{\frac{\theta(a-c)}{\sqrt{f}}}\right]^2 \theta(1-\theta)n^{*T} \\
&= U^A + \frac{(1-\theta)fn^{*T}}{\theta} \\
&> U^A,
\end{aligned}
$$

where the second equation follows from $\theta n^T + n^{*T} = \theta n^A$, the third uses (18), and the fourth comes from (16). As a result, it is established that $U^T > U^A$ unless either $\theta = 0$ or $\theta = 1$. In either of these extreme cases, $U^T = U^A$, i.e., trade has no net effect on welfare. **Q. E. D**

Proposition 2 has a fairly strong implication since θ does not matter for gainfulness of trade except for $\theta = 1$. In other words, positive gains from trade are guaranteed for any $\theta \in (0, 1)$. The reason is as follows. Under free entry, all the private firms' profit vanishes. Therefore, free trade necessarily improves welfare by raising consumer surplus, i.e., there is no welfare loss that comes from profit losses for Home firms.

Nevertheless, there is no net gain in the limiting case with $\theta = 1$. In this case, simple calculations convince us that

$$X^A = X^T = \frac{a - c - \sqrt{f}}{a - c}.$$

That is, the total supply is the same under both autarky and free trade. Therefore, noting that the profit of all firms including the public firm is now zero, welfare equals consumer surplus only. Given this fact and $X^A = X^T$, it naturally follows that welfare is unchanged from autarky to free trade. According to this interpretation, the net effect of trade liberalization is infinitesimally small in developed countries where $\theta \approx 1$, but globalization is a welfare-improving policy for developing countries with a relatively small θ.

5 Trade gains with optimal privatization

In the foregoing arguments, we have ruled out the possibility that θ is optimally chosen to maximize welfare. This section addresses how gains from trade are affected by endo-

geneity of θ. For this purpose, let us prove another useful result:

Lemma 1. *Regardless of free entry among private firms, full nationalization is optimal.*

Proof. Let us begin by deriving the optimal level of θ in the short-run. Differentiating (10) with respect to θ, we have

$$\left.\frac{dU^T}{d\theta}\right|_{n,n^*:fixed} = \frac{-\theta(n^*+1)(2n^*+1)(a-c^2)}{[\theta(n+1)+n^*+1]^3} < 0,$$

from which we can conclude that $\theta = 0$ is optimal. It is obvious that the same result applies to autarky with $n^* = 0$.

Before computing the optimal level of θ in the long-run, we should note that the free entry condition (15) has alternative expressions:

$$\begin{aligned} \theta n + n^* + 1 &= \theta\left(\frac{a-c}{\sqrt{f}} - 1\right) \\ \theta(n+1) + n^* + 1 &= \frac{\theta(a-c)}{\sqrt{f}}. \end{aligned}$$

Substituting these into (17), the Home welfare with free entry can be rewritten as

$$\begin{aligned} U^T &= \frac{f}{2}\left\{\left(\frac{a-c}{\sqrt{f}} - 1\right)^2 + \frac{2[n^*(1-\theta)+1]}{\theta}\right\} - f \\ &= \frac{f}{2}\left(\frac{a-c}{\sqrt{f}} - 1\right)^2 + \frac{f[n^*(1-\theta)+1]}{\theta} - f. \end{aligned}$$

On the other hand, n^* under free entry depends on θ in such a way that

$$\begin{aligned} \frac{dn^*}{d\theta} &= \frac{a-c}{\sqrt{f}} - n - 1 \\ &= \frac{n^*+1}{\theta}, \end{aligned}$$

where the last equation is obtained by rewriting (15). Taking into account this dependence of n^* on θ and differentiating U^T above with respect to θ, we have

$$\left.\frac{dU^T}{d\theta}\right|_{free\ entry} = \frac{\partial U^T}{\partial n^*} \cdot \frac{dn^*}{d\theta} + \frac{\partial U^T}{\partial \theta}$$

$$
\begin{aligned}
&= f\left(\frac{1-\theta}{\theta}\cdot\frac{dn*}{d\theta}-\frac{n^*+1}{\theta^2}\right)\\
&= f\left(\frac{1-\theta}{\theta}\cdot\frac{n^*+1}{\theta}-\frac{n^*+1}{\theta^2}\right)\\
&= -\frac{(n^*+1)f}{\theta}<0,
\end{aligned}
$$

which allows us to state that $\theta = 0$ is optimal under autarky ($n^* = 0$) and free trade ($n^* > 0$). Accordingly, $\theta = 0$ is optimal regardless of free entry and the presence of foreign firms. **Q. E. D.**

Applying Lemma 1 to Propositions 1 and 2, striking results can be established, which are stated without proof:

Proposition 3. *Suppose that θ is optimally determined by the Home government. Then, there is neither gains nor losses from trade regardless of free entry.*

It is almost obvious that $\theta = 0$ is the best policy for welfare maximization because marginal cost pricing prevails and market distortion associated with imperfect competition disappears. But, much information on preference and production technologies is required to implement such a first-best policy and hence it is in reality quite difficult to set $\theta = 0$. In this sense, the seemingly unrealistic implication in Proposition 3 should not be overestimated.

6 Final remarks

Constructing a simple model of mixed oligopoly, we have identified some results on gains from trade in the presence of a public firm. Despite many simplifying assumptions, we believe that the results obtained are of some importance from the theoretical and practical viewpoints. In particular, Propositions 1 and 2 are in sharp contrast to each other since the degree of privatization crucially affects the possibility of trade gainful trade in the short-run whereas it has no substantial impact on gainfulness of trade in the long-run.

However, we need to relax some of the convenient assumptions made in this paper.

One of the biggest assumptions to be modified is that of identically constant marginal cost. Most of the existing literature allows for not only asymmetric costs but also increasing marginal cost, which are more realistic. In addition, we have considered a home market model of mixed oligopoly for analytical convenience. While this model is tractable and makes the results transparent, it neglects the possibility that Home firms export. In order to overcome this difficulty, one has to formulate an integrated market model like Bercena-Ruiz and Garzon (2005) or two-way trade model elaborated by Brander and Krugman (1983). These tasks are left as future research agenda.

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