A Trade Model of Inequality and Varieties

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Abstract

A trade model with many industries featuring monopolistic competition is built to analyze trade patterns in the presence of differences in capital distribution and non-homothetic preferences. We find that, the more unequal country produces a larger number of varieties; Second, the opening to trade will unambiguously increase the number of varieties produced and consumed by any country; Third, the more unequal the partner country, the better. Fourth, a redistributive policy may harm consumers by diminishing the number of varieties consumed. Also, we veriy that trade liberalization may have domestic demand-creating effects so that sectors that did not produce may start producing when exposed to foreign competition.

Keywords: monopolistic competition, inequality, non-homothetic preferences

1 Introduction

2 The Model

This section builds the basic model with non-homothetic preferences and income inequality. There are two countries, Home and Foreign, and M sectors featuring monopolistically competitive with every firm producing a different variety.

Home and Foreign populations have mass L and L^*, respectively.[1] Each individual possesses h units of capital (or efficiency level h) which is given by a distribution function

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[1] Foreign variables and parameters will be denoted by (*).

$f(h)$ with $h \in [h_m \in \infty]$ such that:[2]

$$\int_{h_{min}}^{\infty} f(h)dh = 1 \qquad (1)$$
$$\int_{h_{min}}^{\infty} hf(h)dh = \bar{h}$$

Here, $f(h)$ determines the fraction of population with productivity level h and $\bar{h}$ is the average productivity of the entire population. Thus the total amount of effective labor is given by $L\bar{h}$ and, when wage is w, a consumer with efficiency level h have income $I^h = wh$.

2.1 Consumers

Consumers derive utility level U from the consumption of goods in the following manner:

$$U = \sum_{i=0}^{M} \ln[D_i + g(i)] \qquad (2)$$

where D_i denotes the consumption of differentiated goods of sector i and $g(i) > 1$ is a function with value increasing monotonically in i. The additive separable form of the utility function gives the especial feature we need: goods will only be consumed when there is sufficient income. If the price increases monotonically with the sector index then the god with the lowest index will be consumed first then subsequent goods will be consumed gradually as income increases. This feature give goods a ranking of necessity or taste we need in order to analyze income distribution issues.[3]

The subutility of sector i takes the following Dixit-Stliglitz utility form:

$$D_i = \left(\sum_{k=1}^{n_i} (d_{ki})^{\theta} + \sum_{k^*=1}^{n_i^*} (d_{k^*i})^{\theta} \right)^{\frac{1}{\theta}}, \quad 0 < \theta < 1, \qquad (3)$$

where ki (k^*i) denotes the k-th (k^*-th) differentiated good of sector i produced at Home (Foreign), n_i (n_i^*) is the total number of Home (Foreign) differentiated goods of sector i, d_{ki} denotes Home consumption of Home variety ki and d_{k^*i} denotes Home consumption of Foreign variety k^*. Here, $(1/1-\theta)$>1 is the elasticity of substitution between every

[2] For illustrative purposes, later we specify the cumulative distribution function.
[3] See Fukushima(2008) for a two-sector model.

pair of differentiated goods. The price index of sector i, P_i, takes the form

$$P_i = \left(\sum_{k=1}^{n_i} (p_{ki})^{\frac{\theta}{\theta-1}} + \sum_{k^*=1}^{n_i^*} (p_{k^*i})^{\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}, \tag{4}$$

where p_{ki} and p_{k^*i} denote the prices paid by Home consumers for Home variety ki and Foreign variety k^*i respectively.

We solve utility maximization problem in two steps. First, by solving the subutility maximization problem we obtain the Home demand for Home variety ki and Foreign variety k^*i, respectively:

$$d_{ki} = \left(\frac{p_{ki}}{P_i}\right)^{\frac{1}{\theta-1}} D_i \tag{5}$$

$$d_{k^*i} = \left(\frac{p_{k^*i}}{P_i}\right)^{\frac{1}{\theta-1}} D_i. \tag{6}$$

In the second step consumers maximize utility over the sectors. Note, however, that the demand levels for each sector will be contingent to the level of income. Thus, a consumer with income level h will have the following demand function for sector i goods:

$$D_i^h = \begin{cases} \frac{I^h[\sum_{j=0}^{m(h)} D_j + \sum_{j=0}^{m(h)} g(j)]}{P_i \sum_{j=0}^{m(h)} D_j} - g(i) & \text{if } I^h \geq \frac{P_i g(i) \sum_{j=0}^{m(h)} D_j}{\sum_{j=0}^{m(h)} D_j \sum_{j=0}^{m(h)} D_j} \\ 0 & \text{if } I^h < \frac{P_i g(i) \sum_{j=0}^{m(h)} D_j}{\sum_{j=0}^{m(h)} D_j \sum_{j=0}^{m(h)} D_j}, \end{cases} \tag{7}$$

Where $m(h)$ is the number of sectors consumer h consumes goods from. Note that diferent efficiency levels imply in different number of sectors with woth positive consumption by a consumer and different quantities as well. We assume a large number of sectors to simplify calculations.

Rearranging equations (5) to (7) we obtain the demand function for a Home variety

ki and a Foreign variety k^*i of a consumer with skill level h:

$$d_{ki}^h = \begin{cases} \left(\frac{p_{ki}}{P_i}\right)^{\frac{1}{\theta-1}} \frac{I^h[\sum_{j=0}^{m(h)} D_j + \sum_{j=0}^{m(h)} g(j)]}{P_i \sum_{j=0}^{m(h)} D_j} - g(i) & \text{if } I^h \geq \frac{P_i g(i) \sum_{j=0}^{m(h)} D_j}{\sum_{j=0}^{m(h)} D_j \sum_{j=0}^{m(h)} D_j} \\ 0 & \text{if } I^h < \frac{P_i g(i) \sum_{j=0}^{m(h)} D_j}{\sum_{j=0}^{m(h)} D_j \sum_{j=0}^{m(h)} D_j}, \end{cases} \tag{8}$$

$$d_{k^*i}^h = \begin{cases} \left(\frac{p_{k^*i}}{P_i}\right)^{\frac{1}{\theta-1}} \frac{I^h[\sum_{j=0}^{m(h)} D_j + \sum_{j=0}^{m(h)} g(j)]}{P_i \sum_{j=0}^{m(h)} D_j} - g(i) & \text{if } I^h \geq \frac{P_i g(i) \sum_{j=0}^{m(h)} D_j}{\sum_{j=0}^{m(h)} D_j \sum_{j=0}^{m(h)} D_j} \\ 0 & \text{if } I^h < \frac{P_i g(i) \sum_{j=0}^{m(h)} D_j}{\sum_{j=0}^{m(h)} D_j \sum_{j=0}^{m(h)} D_j}, \end{cases} \tag{9}$$

(10)

Consumers will start demanding the good of the first sector. As income increases and reaches a certain level, she will start to demand the next good, now deviding the extra income equally among the two sector. Also, note that there is a consumer that will have income level exactly equal to the point where she starts consuming a new good. This is the *marginal consumer* of the variety in question and it is appropriate to give a definition:

$$h_{mg}^i \equiv \left\{ h^i \in [h_{min}, \infty] \quad | \quad h^i = \frac{P_i g(i) \sum_{j=0}^{m(h)} D_j}{[\sum_{j=0}^{m(h)} D_j + \sum_{j=0}^{m(h)} g(j)] w} \right\}. \tag{11}$$

The marginal consumer indicates the minimum level of h necessary to consume a certain variety i. We derive now the aggregate demands for goods of sector i. But first, let us derive the following relations for the consumer of variety i. Defining $\bar{h_i}$ as the average efficiency level of consumers of variety of sector i and δ_i as the fraction for total population of those cosumers, we have:

$$\delta_i \times \bar{h_i} \equiv \int_{h_{mg}}^{\infty} h f(h) dh, \qquad \delta_i \equiv \int_{h_{mg}}^{\infty} h dh.$$

Using the above definitions and equations (8) to (10), the Home aggregate demand for

for Home variety ki, d_{ki} , and aggregate demand for Foreign variety k^*i, d_{k^*i}, are derived:

$$d_{ki} = \left(\frac{p_{ki}}{P_i}\right)^{\frac{1}{\theta-1}} g(i)L\left[\frac{\delta_i \times \bar{h_i}}{h^i_{mg}} - \delta_i\right] \tag{12}$$

$$d_{k^*i} = \left(\frac{p_{k^*i}}{P_i}\right)^{\frac{1}{\theta-1}} g(i)L\left[\frac{\delta_i \times \bar{h_i}}{h^i_{mg}} - \delta_i\right]. \tag{13}$$

Thus under a given skill distribution curve, the aggregate demand can be completely defined.

2.2 Production

Now let us turn to the supply side. As we have assumed, Differentiated goods are produced in monopolistically competitive sectors with production requiring a fixed amount μ_i, that increases with the sector index, and a variable amount β of labor similar across sectors and countries.[4] Countries share the same production technology. Then, the pricing rule for Home and Foreign, respectively, is given by:[5]

$$p_{ki} = p^*_{ki} = \frac{\beta w}{\theta}, \quad \text{and} \quad p_{k^*i} = p^*_{k^*i} = \frac{\beta w^*}{\theta}.$$

Given the above pricing rule, the profits of a Home firm producing variety ki and a Foreign firm producing variety k^*i are denoted, respectively, by:

$$\pi_{ki} = (1-\theta)\frac{\beta w}{\theta}(d_{ki} + d^*_{ki}) - w\mu_i,$$
$$\pi_{k^*i} = (1-\theta)\frac{\beta w^*}{\theta}(d_{k^*i} + d^*_{k^*i}) - w^*\mu_i.$$

Assuming free entry and exit of firms in the long run, we obtain the following zero-profit conditions:

$$(1-\theta)\frac{\beta w}{\theta}(d_{ki} + d^*_{ki}) - w\mu_i = 0, \tag{14}$$

$$(1-\theta)\frac{\beta w^*}{\theta}(d_{k^*i} + d^*_{k^*i}) - w^*\mu_i = 0. \tag{15}$$

Note that some sectors may have too large fixed cost coefficient μ^i so that it may

[4] It is possible to allwo β to vary across sectors. Some interesting results can be achieved.

[5] This paper does not consider trade costs, but an extension of the model could include trade barriers.

not be profitable. In that case we will have a "marginal sector" which is the last sector that allows firms to have non-negative profits. With the above conditions we can start our analysis on inequality. In the next section we discuss the effects of skill inequalities in autarky.

3 The Autarky Economy

As a benchmark, let us consider the case in which there is no trade between Home and Foreign.[6] Given the symmetry of firms in each sector, we are able to calculate the price index $P_i = wn_i^{\frac{\theta-1}{\theta}}\beta/\theta$ and the marginal consumer of sector i:

$$h^i_{mg} = \frac{n^{\frac{\theta-1}{\theta}}\beta g(i)\sum_{j=0}^{m(h)} D_j}{\theta[\sum_{j=0}^{m(h)} D_j + \sum_{j=0}^{m(h)} g(j)]}.$$

Note that, if the number of firms in equilibrium is sufficiently high, the marginal consumer may be smaller than the support of the distribution function, that is, $h_{mg} < h_{min}$, implying that all consumers are able to consume goods of sector i. First, we analyze the case in which there are people that consume goods of sector i and others that do not consume in equilibrium, that is, $h_{mg} > h_{min}$, then we proceed to the case in which everyone consumes.

3.1 The Mixed Case

Using the demand equations, the price index and the marginal consumer we have derived we are able to obtain the equilibrium conditions. However, the magnitude of $\bar{s_R}$ and δ_R can only be determined with a specific distribution function. For the purpose of illustration we consider the following Pareto distribution function:

$$f(h; h_{min}, k) = \frac{kh^h_{min}}{h^{k+1}}, \quad \text{with} \quad h_{min} > 0, \quad k > 1$$

where h_{min} is the support of the function (minimum skill) and k is a parameter denoting skill inequality.[7] Now it is possible to calculate the average skill and the fraction of

[6] We only analyze the case of Home, but the Foreign case is analogous.

[7] If $k = 1$ there is complete inequality and if $k = \infty$ there is perfect equality. Note that when there is no redistributive policy, skill inequality and income inequality are equivalent.)

consumers, and the marginal consumer of sector i:

$$\delta_i \times \bar{h}_i = \frac{kh_{min}^k}{(k-1)h_{mg}^{k-1}}, \qquad \delta_R = \frac{h_{min}^k}{h_{mg}^k}. \tag{16}$$

The equilibrium number of firms is obtained by the following three equations:

$$n_i(h_{mg}) = \left[\frac{Lh_{min}^k g(i)(1-\theta)\beta}{(k-1)\mu_i\theta h_{mg}^k}\right]^\theta \tag{17}$$

$$h_{mg}^i = \frac{n^{\frac{\theta-1}{\theta}}\beta g(i)\sum_{j=0}^{m(h)} D_j}{\theta[\sum_{j=0}^{m(h)} D_j + \sum_{j=0}^{m(h)} g(j)]} \tag{18}$$

$$h_{mg}^i > h_{min}. \tag{19}$$

Equations (17) and (18) constitute loci of which intersection uniquely determines the equilibrium number of firms. However, the intersection point is eligible as an equilibrium point only if it is stable and satisfies condition (19). Figure 1 denotes a case in which the intersection point e is eligible as an equilibrium point because $h_{mg}^e > h_{min}$ and the point is stable since curve n_i cuts curve h_{mg} from above as n_i increases.

[Figure 1 around here]

With lower levels of inequality, particularly if $k > 1/(1-\theta)$, curve $n(s_m)$ will cut curve $s_m(n)$ from below making the intersection point unstable. Also, there may be cases in which the intersection point does not satisfy condition (19).

3.2 The non-Mixed Case

Next we examine the case in which all consumers are able to consume varieties of sector i. In such cases no marginal consumer, that is, even the consumer with the lowest skill level h_{min} consumes a positive amount of the differentiated products of that sector. The equilibrium conditions are:

$$h_{mg}^i(n_i) = \frac{Lh_{min}g(i)k(1-\theta)\beta}{(k-1)[\mu_i\theta n_i^{1/\theta} + g(i)\beta(1-\theta)L]} \tag{20}$$

$$h_{mg}^i = \frac{n^{\frac{\theta-1}{\theta}}\beta g(i)\sum_{j=0}^{m(h)} D_j}{\theta[\sum_{j=0}^{m(h)} D_j + \sum_{j=0}^{m(h)} g(j)]} \tag{21}$$

$$h_{mg}^i < h_{min}. \tag{22}$$

The intersetion of points of (20) and (21) are candidates for equilibrium points. They will generally intersect twice. Note, however, that only one point is stable. As in the previous case, there is a maximum value the marginal consumer can assume, if she existed, so that the stable point may be eligible as an equilibrium point. This is illustrated in Figure 2.

[Figure 2 around here]

Figure 2 depicts the case with two intersection points. Point a, however, is not eligible as an equilibrium point since it is not stable, leaving e as the only equilibrium point since it lies below the h_{min} line. There will be cases that no points will exist or satisfy condition (22).

3.3 Autarky Equilibrium

We have seen the two cases separately, but in order to make a complete analysis of the equilibrium it is necessary to consider both cases at once. From conditions (17) and (20) notice that curve $n_i(h_{mg})$ will necessarily intersect the descending curve $h_{mg}(n_i)$ twice: when $h_{mg} = h_{min}$ and when $h_{mg} < h_{min}$.[8] With trade liberalization it is possible to have several different equilibrium configurations. A typical equilibrium is depicted in Figure 3, when $k < 1/(1-\theta)$.

[Figure 3 around here]

Considering only stable equilibria, we note that when $k < 1/(1-\theta)$ an equilibrium always exists. When $k > 1/(1-\theta)$, however, the mixed equilibrium (with not eveybody consuming goods of sector i) is not stable.

Now we focus on the differences in inequality levels and labor endowment and the effects on the autarkic equilibrium. From equation (20) we know that a higher level of inequality, that is, a lower k, will cause a parallel upward shift in the descending line. Simultaneously, curve n_i will also shift upward so as to intersect with the descending line at $h_{mg} = h_{min}$. In the $k < \frac{1}{1-\theta}$ case, the equilibrium number of firms increases and the equilibrium configuration may change from mixed to non-mixed.

More unequal economies are likely to have a higher larger number of firms since a larger fraction of people have higher wages, which causes an increase in demand for

[8]See Appendix A1.

varieties. In turn, higher demand increases the number of varieties and, consequently, the wage of the marginal consumer decreases, creating more demand. When $k > \frac{1}{1-\theta}$ we have seen that the equilibrium number of firms is determined by the intersection of the h_{mg} curve and the descending curve h_{mg}. Since the descending curve shifts upward the number of firms unambiguously increases with a lower k. The above result is summarized as follows.

Lemma 1. *In autarky, more unequal countries have a larger number of firms.*

As for labor endowment, from (20) and (17) we know that a larger population will cause the descending line to rotate upward, and that curve n_i will shift upward so as to intersect with the descending curve when $h_{mg} = h_{min}$. As a result the number of varieties produced will increase in any equilibrium configuration.

Lemma 2. *In autarky, countries with larger labor mass L will have a larger number of firms.*

Figure 4 depicts the case in which $k < \frac{1}{1-\theta}$.

[Figure 4 around here]

As can be verified from the above results, the equilibrium number of firms can change drastically even with a small change in parameters, suggesting that trade liberalization can cause great changes in the production structure of countries. The next section examines these changes.

4 The Free Trading Economy

In this section the effects of trade liberalization and income inequality are examined. We assume that Home and Foreign have the same production technologies and trade freely. the price of the homogeneous good is w, and the price of any variety is $\beta w/\theta$. From the symmetry of firms, the price index can be derived:

$$P_i = (n_i + n_i^*)^{\frac{(\theta-1)}{\theta}} \frac{\beta w}{\theta} = N_i^{\frac{(\theta-1)}{\theta}} \frac{\beta w}{\theta},$$

where N_i is the total number of varieties of sector i and is consisted of the sum of n_i Home varieties and n_i^* Foreign varieties.

As has been verified, there are many equilibrium configurations and, even with small differences, Home and Foreign may have different production patterns under autarky. As such, the opening to trade may change them considerably. Our analysis can focus on the economy of one country only to see how the patterns of production and consumption change.

First, we examine how the equilibrium conditions in one country change due to the opening to trade, then, we analyze the change in the number of varieties produced before and after trade. Denoting free trade variables with T, we set the number of Foreign varieties in equilibrium as a multiple of Home varieties so that $n_i^{T*} = \alpha n_i^T$ (with $\alpha \geq 0$). Then $N^T = (1+\alpha)n_i^T$. From the above analysis, it is possible to conclude that, as $(1+\alpha) > 1$, with the opening to trade the n_i curve shifts upward, the marginal consumer curve shifts inward, and the descending line rotates outward so as to intersect with the n_i curve when $h_{,g} = h_{min}$. We verify that the equilibrium number of Home firms unambiguously increases with trade no matter the initial equilibrium configuration.

Similar results can be found for high levels of k since in the non-mixed stable equilibrium the number of varieties produced will always increase. Then we derive the following result:

Proposition 1. *The opening to trade will unambiguously increase the number of varieties produced and consumed by any country.*

The opening to trade augments the number of varieties consumers can purchase, which reduces the price index. As a consequence, the skill level of the marginal consumer decreases and a greater fraction of consumers start purchasing domestic and foreign varieties, which increases the demand and the number of varieties produced in both countries. It is also possible to deduce the following:

Lemma 3. *Suppose two countries trade freely with each other. Then the higher the number of foreign firms the higher the number of domestic firms.*

Lemmas 1 and 2 imply that countries with higher levels of inequality or larger labor mass have unambiguously a larger number of firms. Thus the following result can be directly derived:

Proposition 2. *The number of firms producing varieties in a country is higher the higher the level of inequality or the size of the trading partner.*

The intuitiveness of this result can be illustrated in the following way. When two countries have equal amount of effective labor, the more unequal country will have a larger portion of rich population and, consequently, higher demand for varieties. Thus, the number of varieties produced is larger. The same applies for countries with different labor endowment.

Note that under this analysis, there maybe cases in which a country does not have a sector active before trade, but after trade liberalization it becames active. Although production was not profitable before trade due to technological or demand-side constraints, the opening to trade makes more varieties available, reducing the price index and creating demand. As a result, more competition is able to "create" demand enough to activate home firms.

5 Redistributive Policy

In this section we analyze the effects of redistributive policies on the economy. Suppose the government redistributes income so as to increase k (decrease inequality) while keeping the average skill $\bar{h}$ unchanged. Since labor mass L is constant, the support of the function h_{min} has to increase. If the inequality index is improved to the level $k^I > k$ the government has to distribute income to the poorest consumers so that the income of the poorest consumer be equivalent to $wh^I_{min} > wh_{min}$. Thus the following holds:

$$\frac{ks_0}{k-1} = \frac{k^I s^I_o}{k^I - 1}. \tag{23}$$

From the equilibrium conditions we verify that the new inequality index will shift the n_i curve upward so that one of the intersections with the descending curve lies on the h^I_{min} line. It is also possible to verify that the new n^I_i curve will intersect the old n_i curve somewhere below the h_{min} line. This is depicted in Figure 5 below.

[Figure 5 around here]

Figure 5 depicts case of mixed equilibriua. In this case the new equilibrium number of firms n_i^I decreases due to a redistributive policy. In the case of non-mixed equilibria there will be no change in the number of firms since the descending curve as well as the h_{mg} line will not move. Our result is summarized as follows:

Proposition 4. *A redistributive policy that diminishes inequality while keeping average productivity constant may decrease the number of firms in equilibrium.*

6 Concluding Remarks

This paper builds a trade model with non-homothetic preferences and monopolistic competition. There are many monopolistically competitive sectors with differentiated products. Consumers derive income solely from labor, which skill level is distributed unevenly among the population.

The main results are: First, in autarky, the more unequal country produces a larger number of varieties; Second, the opening to trade will unambiguously increase the number of varieties produced and consumed by any country; Third, the more unequal the partner country, the better. Fourth, a redistributive policy may harm consumers by diminishing the number of varieties.

This paper provided through a very simple framework in a systematic way to deal with inequality and trade. Although simplifications are done to make the model tractable it does not lose its qualitative features and bring some interesting counterintuitive results. The "demand-creating" feature that is caused by the non-homotheticity of the utility function augments the gains from trade. Further research could be done to iclude more general distribution functions. Also, political issues could be included to endogenize income distribution.

Appendix

A1

To be written

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