Firm Heterogeneity, Technology Utilization, and International Fragmentation

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Abstract

We study heterogeneous firm's optimal decision on technology utilization and the extent of international fragmentation in a monopolistic competition, general equilibrium North-South trade model. The production of the manufactured goods requires the inputs of intermediate goods, which can be produced either in the North or South. We show that firms may not utilize all available intermediate goods and if the elasticity of substitution between intermediate goods in the manufacturing is smaller than the elasticity of substitution in consumption between varieties of the manufactured good, the firm with a lower capacity to utilize the intermediate goods uses only the subset of the intermediate goods that the firm with a higher capacity. When they have an option of fragmenting their production process and shifting a part to the South, firms with higher capacity tend to do so, while firms with lower capacity remain in the North.

Preliminary and incomplete.

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1 Introduction

The issue of multinational firm's optimal technology adoption and its choice of organizational form in the global market has recently gained attention in the international trade literature. Acemoglu, Antràs, and Helpman (2007) analyze the relationship between contract incompleteness and technology adoption in relation to the complementarity between intermediate inputs. Ederington and McCalman (2004) consider technology diffusion process, giving a theory as to why firms with different productivities coexist simultaneously in an industry. Furusawa and Sato (2007) investigate the adoption of new technology by heterogeneous firms and discuss its relationship with factor endowments.

This paper takes a different angle to study heterogeneous firms' decision on technology utilization and the degree of international fragmentation of the production process. We develop a monopolistic-competition general equilibrium North-South trade model in which labor and capital are two primary factors used to produce intermediate inputs. Final good production of any variety of a manufactured good require a set of the intermediate input varieties which differ in their capital-intensity in production. Firms with different capacities to utilize the intermediate goods choose in general different sets of intermediate goods that they utilize.

In the environment where capital is relatively abundant in both countries and hence the production costs of capital-intensive intermediate goods are lower than those of labor-intensive intermediate goods, we first examine the choice of technology utilization by firms that differ in their capacity to utilize the intermediate goods. We find that firms may not utilize all available intermediate goods and that the relationship between the utilization level and the capacity depends on whether or not the manufactured-good varieties are more substitutable between themselves than the intermediate goods in the production of the manufactured good. We focus on the relatively more realistic case where the elasticity of substitution between intermediate goods in the manufacturing is smaller than the elasticity of substitution in consumption between varieties of the manufactured good. In that case, the firm with a lower capacity to utilize the intermediate goods uses only the subset of the intermediate goods that the firm with a higher capacity. When firms in the manufactured good industry are given an option to fragment their production process and shift part of the intermediate-goods production to the

South, firms with high capacities do so, while firms with low capacities remain in the South.

2 Model

We consider a general equilibrium model of North-South trade. Let $L_N(L_S)$ and $K_N(K_S)$ be the northern (southern) endowments of labor and capital, respectively. There are two goods produced and consumed in this economy. One is the manufactured good X with differentiated varieties, and the other is the numeraire good Y produced competitively with a constant-returns-to-scale technology.

There are n_N consumers and n_S consumers in the North and South, respectively, and all of $n = n_N + n_S$ consumers share the same preferences. Letting m and $\beta > 1$ denote the mass of varieties and the elasticity of substitution in the industry X, we specify a representative consumer's utility function as

$$U = \log \left[\int_0^m X(j)^{\frac{\beta-1}{\beta}} dj \right]^{\frac{\beta}{\beta-1}} + Y,$$

where while $X(j)$ and Y denote the consumption levels of the j-th variety of good X and good Y, respectively. It is easy to show that with this type of quasi-linear utility function, each consumer spend 1 on good X and the rest of her income on good Y. Aggregate demands for variety j equals

$$X(j) = np(j)^{-\beta} P^{\beta-1}, \tag{1}$$

where $P \equiv \left[\int_0^m p(j)^{1-\beta} dj \right]^{\frac{1}{1-\beta}}$ denotes the price index in the industry X.

Good Y is produced from labor and capital with the production function

$$f_Y(L, K) = \left(\frac{L}{1-\gamma} \right)^{1-\gamma} \left(\frac{K}{\gamma} \right)^{\gamma},$$

whose corresponding cost function is given by $c_Y(w, r) = w^{1-\gamma} r^{\gamma}$ with w and r representing the wage rate and rental rate, respectively. Since good Y is the numeraire, we have $c_Y(w, r) = 1$ and hence $w = r^{\frac{\gamma}{\gamma-1}}$ in the country that produces good Y.

Good X is produced from the varieties of intermediate goods, or tasks, each of which is a point in $[0,1]$. Firms, however, differ in their capacity to utilize these intermediate goods. Upon the entry to the industry X, firms hire f_e units of labor and engage in R&D to develop a variety and to raise their capacities to utilize the intermediate goods. This R&D process is characterized by a lottery in which each firm pays wf_e to pick a capacity θ from the probability distribution whose density function is h defined on $[\bar{\theta},1]$, where $\gamma < \bar{\theta} < 1$. The restriction $\gamma < \bar{\theta}$ suggests that good X is likely to be capital-intensive.

Firm j, which is a sole producer of variety j and has selected the capacity $\theta(j)$ can only utilize the intermediate goods whose indices are less than or equal to $\theta(j)$. We assume that firms incur the fixed labor cost wf to use each intermediate goods, so firms may not use all intermediate goods that they can utilize. Firm j utilizes intermediate goods that are a subset of $[0,\theta(j)]$ to produce good X. As is evident soon, firms always choose an interval of intermediate goods, so we can specify the production function of good $X(j)$ as

$$X(j) = \left[\int_{\theta_l(j)}^{\theta_h(j)} x(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution and $[\theta_l(j),\theta_h(j)] \subset [0,\theta(j)]$.

Production function of intermediate good $i \in [0,1]$ is given by

$$x(i) = \left(\frac{L}{1-i}\right)^{1-i}\left(\frac{K}{i}\right)^{i},$$

whose corresponding cost function is given by

$$c(w,r,i) = w^{1-i}r^{i} = w\left(\frac{r}{w}\right)^{i}.$$

We focus on the case where $w/r > 1$ in both countries, so $c(w,r,i)$ is decreasing in i. Therefore, firm j utilize its most capital-intensive intermediate good, i.e., $\theta_h(j) = \theta(j)$.

3 Technology Utilization without FDI

This section considers firms' choice of technology utilization when FDI of intermediate goods is not an option. As we have mentioned, even though the technology to produce the good X from intermediate goods is a love-of-variety type, firms may not utilize every intermediate good that they are capable to utilize due to the existence of fixed costs. We first consider the industry equilibrium and show the characteristics of technology utilization depending on the firms' capacities. Then we close the model considering the general equilibrium of the North-South trade.

A firm that selected the capacity θ, which we henceforth call firm θ rather than firm j with the capacity θ, chooses which and how much of intermediate goods that it produces in order to make the final good X. Since $\theta_h(\theta) = \theta$, firm θ's problem can be written as

$$\begin{aligned} \min_{\theta_l, x} \quad & \int_{\theta_l}^{\theta} c(w,r,i)x(i)di + (\theta - \theta_l)wf \\ \text{s.t.} \quad & \left[\int_{\theta_l}^{\theta} x(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \geq X. \end{aligned}$$

Focusing on the case where the problem yields interior solutions, we derive the first-order conditions as

$$\begin{aligned} c(w,r,i) &= \lambda X^{\frac{1}{\sigma}} x(i)^{-\frac{1}{\sigma}}, \\ c(w,r,\theta)x(\theta_l) + wf &= \frac{\sigma}{\sigma-1} \lambda X^{\frac{1}{\sigma}} x(\theta_l)^{\frac{\sigma-1}{\sigma}}, \end{aligned}$$

where λ is the Lagrange multiplier of the problem. The Lagrange multiplier λ in this case is the marginal cost of producing good X and is given by

$$\lambda = \left[\int_{\theta_l}^{\theta} c(w,r,i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \equiv \tilde{c}(w,r,\theta_l,\theta).$$

Then, we have

$$x(i) = \tilde{c}(w,r,\theta_l,\theta)^{\sigma} c(w,r,i)^{-\sigma} X$$

and

$$c(w, r, \theta_l)x(\theta_l) = (\sigma - 1)wf,$$

So we obtain

$$\tilde{c}(w, r, \theta_l, \theta)^{\sigma} c(w, r, \theta_l)^{1-\sigma} X = (\sigma - 1)wf. \tag{2}$$

Firm θ's total costs can be calculated as

$$\begin{aligned} &\int_{\theta_l}^{\theta} \tilde{c}(w, r, \theta_l, \theta)^{\sigma} c(w, r, i)^{1-\sigma} X di + (\theta - \theta_l)wf \\ &= \tilde{c}(w, r, \theta_l, \theta)X + (\theta - \theta_l)wf \end{aligned}$$

Firms maximize their individual profits with this cost function. It is known that every firm engages in the mark-up pricing such that $p(\theta) = [\beta/(\beta - 1)]\tilde{c}(w, r, \theta_l(\theta), \theta)$. Here, we indicates that firm θ chooses $\theta_l(\theta)$ in equilibrium. Then it follows from the definition of the price index that

$$\begin{aligned} P &= \left[\int_{\underline{\theta}}^{1} mp(\theta)^{1-\beta} h(\theta) d\theta\right]^{\frac{1}{1-\beta}} \\ &= m^{\frac{1}{1-\beta}} \frac{\beta}{1-\beta} \bar{c}(w, r), \end{aligned}$$

where

$$\bar{c}(w, r) = \left[\int_{\underline{\theta}}^{1} \tilde{c}(w, r, \theta_l(\theta), \theta)^{1-\beta} h(\theta) d\theta\right]^{\frac{1}{1-\beta}}.$$

Substituting these results into (1) to obtain

$$X(\theta) = \frac{(\beta - 1)n}{\beta m} \tilde{c}(w, r, \theta_l(\theta), \theta)^{-\beta} \bar{c}(w, r)^{\beta-1}. \tag{3}$$

Thus, in the industry equilibrium for $\bar{c}(w, r)$ being given, firm θ chooses $\theta_l(\theta)$ so as to satisfy

$$\frac{(\beta - 1)n}{\beta m} \tilde{c}(w, r, \theta_l(\theta), \theta)^{\sigma-\beta} c(w, r, \theta_l(\theta))^{1-\sigma} \bar{c}(w, r)^{\beta-1} = (\sigma - 1)wf. \tag{4}$$

Equation (4) allows us to compare the technology utilization choice between firms with different capacities. Consider two firms with capacities θ_1 and θ_2 with $\theta_1 < \theta_2$. Substituting θ_1 and θ_2 for θ in (4) and dividing the resulting equation for θ_1 by the other, we obtain

$$\frac{c(w,r,\theta_l(\theta_1))}{c(w,r,\theta_l(\theta_2))} = \left[\frac{\tilde{c}(w,r,\theta_l(\theta_1),\theta_1)}{\tilde{c}(w,r,\theta_l(\theta_2),\theta_2)}\right]^{\frac{\sigma-\beta}{\sigma-1}}.$$

Since firm θ_2 has a higher capacity and hence more efficient than firm θ_1, we have $\tilde{c}(w,r,\theta_l(\theta_1),\theta_1) > \tilde{c}(w,r,\theta_l(\theta_2),\theta_2)$. Thus, $c(w,r,\theta_l(\theta_1)) > c(w,r,\theta_l(\theta_2))$ and hence $\theta_l(\theta_1) < \theta_l(\theta_2)$ if $\sigma > \beta$, and vice versa. If σ is large, firms have less incentive to expand the range of intermediate goods in order to lower the production costs. Therefore, the smallest index of the intermediate good that the firm with a higher capacity chooses is greater than that of the firm with a lower capacity. If β is large, on the other hand, the more efficient firm with a higher capacity attracts large demands, which raises the benefit of expanding the range of intermediate good. Thus, the smallest index of the intermediate good that the firm with a higher capacity chooses is smaller than that of the less efficient firm. We record this finding as a proposition.

Proposition 1 *If the elasticity of substitution between intermediate goods in the manufacturing is greater than the elasticity of substitution in consumption between varieties of the manufactured good, the smallest index of the intermediate good that the firm with a higher capacity chooses is greater than that of the firm with a lower capacity. If the ranking of the elasticity is opposite, the set of intermediate goods that the firm with a lower capacity utilizes is a proper subset of the one that for the firm with a higher capacity.*

We think that in most manufacturing industries, the substitutability between final-goods varieties is greater than the substitutability between intermediate goods in production. Thus, we henceforth concentrate on the case where $\sigma < \beta$ so that $[\theta_l(\theta_1), \theta_1] \subset [\theta_l(\theta_2), \theta_2]$ when $\theta_1 < \theta_2$.

Now, we turn to the general equilibrium analysis to determine factor prices and others. We begin with the South where only good Y is produced. We then consider the equilibrium in the North where both good X and Y, as well as intermediate goods for the production of good X, are produced.

In the absence of FDI, the South only produces good Y from its endowment of labor and capital, so the ratio of capital demands to labor demands in the production of good Y equals the capital-labor endowment ratio. It follows from $c_Y(w,r) = w^{1-\gamma}r^{\gamma} = 1$ that the unit labor requirement and unit capital requirement for the production of good Y are given by

$$\frac{\partial c_Y}{\partial w} = \frac{1-\gamma}{w}, \ \frac{\partial c_Y}{\partial r} = \frac{\gamma}{r},$$

so we have $[\gamma/(1-\gamma)](w/r) = K_S/L_S$. Using $c_Y(w,r) = 1$, we obtain

$$w_S = \left(\frac{1-\gamma}{\gamma}\right)^{\gamma}\left(\frac{K_S}{L_S}\right)^{\gamma}, \ r_S = \left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1}\left(\frac{K_S}{L_S}\right)^{\gamma-1}.$$

The South produces $[L_S/(1-\gamma)]^{1-\gamma}(K_S/\gamma)^{\gamma}$ units of good Y, which equals its national income $w_SL_S + r_SK_S$, exporting n_S units of good Y in exchange for good X.

The North demands $w_NL_N + r_NK_N - n_N$ units of good Y. Since n_S units of good Y is imported from the South, the North produces $w_NL_N + r_NK_N - n$ units of good Y, demanding $(r_N^{\frac{\gamma}{\gamma-1}}L_N + r_NK_N - n)\gamma/r_N$ units of capital.

In the good X industry, the mass of the firms m is determined so that the expected profit from the R&D, or the lottery of selecting the capacity, equals zero. It follows from (1) that firm θ's revenue can be written as

$$r(\theta) = n\left(\frac{p(\theta)}{P}\right)^{1-\beta} = \frac{n}{m}\left[\frac{\tilde{c}(w,r,\theta_l(\theta),\theta)}{\bar{c}(w,r)}\right]^{1-\beta}.$$

Due to the simple mark-up pricing behavior of the firm, its profits are given by

$$\pi(\theta) = \frac{n}{\beta m}\left[\frac{\tilde{c}(w,r,\theta_l(\theta),\theta)}{\bar{c}(w,r)}\right]^{1-\beta} - [\theta - \theta_l(\theta)]wf.$$

The free entry condition that the expected profits from the lottery equals zero is derived as

$$\begin{aligned} \int_{\underline{\theta}}^{1} \pi(\theta)h(\theta)d\theta &= wf_e \\ \frac{n}{\beta m} - \int_{\underline{\theta}}^{1}[\theta - \theta_l(\theta)]wfh(\theta)d\theta &= wf_e. \end{aligned}$$

Since $w = r^{\frac{\gamma}{\gamma-1}}$, which is derived from $c_Y(w, r) = 1$, we obtain the free entry condition as

$$m = \frac{n}{\beta r_N^{\frac{\gamma}{\gamma-1}} \{f_e + f \int_{\underline{\theta}}^1 [\theta - \theta_l(\theta)] h(\theta) d\theta\}}. \tag{5}$$

Next, we derive the capital demands in the good X industry to determine the factor prices. Recall that firm θ's total costs are

$$C(w, r, X, \theta) = \tilde{c}(w, r, \theta_l(\theta), \theta) X + [\theta - \theta_l(\theta)] w f. \tag{6}$$

Since a infinitesimal change in r does not affect the total costs through a resulting change in $\theta_l(\theta)$ (the envelope theorem), we have

$$\begin{aligned} \frac{\partial C}{\partial r}(w, r, X, \theta) &= \frac{\partial \tilde{c}}{\partial r}(w, r, \theta_l(\theta), \theta) X \\ &= \tilde{c}(w, r, \theta_l(\theta), \theta)^{\sigma} \left[\int_{\theta_l(\theta)}^{\theta} \frac{i}{r} c(w, r, i)^{1-\sigma} di \right] X. \end{aligned}$$

Since the integration by parts gives us

$$\int_{\theta_l(\theta)}^{\theta} i c(w, r, i)^{1-\sigma} di = \frac{\theta c(\theta)^{1-\sigma} - \theta_l(\theta) c(\theta_l(\theta))^{1-\sigma} - \tilde{c}(\theta_l(\theta), \theta)^{1-\sigma}}{(1-\sigma)\log(r/w)},$$

where we suppress the arguments for factor prices from the functions to simplify the expression, we substitute (3) into the expression above to obtain

$$\frac{\partial C}{\partial r}(w, r, X(\theta), \theta) = \frac{(\beta - 1) n \bar{c}^{\beta-1} \tilde{c}(\theta_l(\theta), \theta)^{\sigma-\beta} [\theta c(\theta)^{1-\sigma} - \theta_l(\theta) c(\theta_l(\theta))^{1-\sigma} - \tilde{c}(\theta_l(\theta), \theta)^{1-\sigma}]}{\beta m r (1-\sigma)\log(r/w)}.$$

Or, defining

$$\Omega(w, r, \theta) = \tilde{c}(\theta_l(\theta), \theta)^{\sigma-\beta} [\theta c(\theta)^{1-\sigma} - \theta_l(\theta) c(\theta_l(\theta))^{1-\sigma} - \tilde{c}(\theta_l(\theta), \theta)^{1-\sigma}]$$

and using $w = r^{\frac{\gamma}{\gamma-1}}$, we write

$$\begin{aligned} \frac{\partial C}{\partial r}(w, r, X(\theta), \theta) &= \frac{(\beta-1)n\bar{c}(w,r)^{\beta-1}\Omega(w,r,\theta)}{\beta m r(1-\sigma)\log(r/w)} \\ &= \frac{(\beta-1)(1-\gamma)n\bar{c}(r^{\frac{\gamma}{\gamma-1}}, r)^{\beta-1}\Omega(r^{\frac{\gamma}{\gamma-1}}, r, \theta)}{\beta m r(1-\sigma)\log r}. \end{aligned}$$

Notice that $w/r = r^{\frac{1}{\gamma-1}} > 1$ implies $r < 1$ (and $w > 1$), so the last expression is positive as expected.

Now, the capital market clearing condition can be written as

$$\int_{\underline{\theta}}^{\theta} \frac{\partial C}{\partial r}(w_N, r_N, \theta) m h(\theta) d\theta + \frac{\gamma}{r}\left(r_N^{\frac{\gamma}{\gamma-1}} L_N + r_N K_N - n\right) = K_N,$$

which can be rewritten as

$$\frac{(\beta-1)(1-\gamma)n\bar{c}(r_N^{\frac{\gamma}{\gamma-1}}, r_N)^{\beta-1}\int_{\underline{\theta}}^{\theta}\Omega(r^{\frac{\gamma}{\gamma-1}}, r, \theta)h(\theta)d\theta}{\beta(1-\sigma)\log r_N} + \gamma\left(r_N^{\frac{\gamma}{\gamma-1}} L_N - n\right) = (1-\gamma) r_N K_N. \quad (7)$$

The free entry condition (5) and capital market clearing condition (7) determine m and r, and consequently determine other endogenous variables in the general equilibrium.

4 International Fragmentation of Production Processes

Some firms in the good X industry may shift (at least) part of the production process in the South if that option is given. This section shows that firms with high capacities shift the production of labor-intensive intermediate goods to the South, while firms with low capacities remain in the North.

Given the assumption that $w_N/r_N > w_S/r_S > 1$, it is easy to see $c(w_N, r_N, \theta) > (<)c(w_S, r_S, \theta)$ if and only if $\theta < (>)\gamma$, i.e., the production costs of the labor-intensive intermediate good are lower in the South than in the North, and vice versa, as Figure 1 shows. Let $\hat{\theta}$ be such that $\theta_l(\hat{\theta}) = \gamma$. Then, under the assumption that $\sigma < \beta$, we have $\theta_l(\theta) > \gamma$ for $\underline{\theta} < \theta < \hat{\theta}$ and $\theta_l(\theta) < \gamma$ for $\theta > \hat{\theta}$. Since the production costs are lower for the intermediate goods whose capital intensity, measured by the share parameter in the production function, is smaller than γ, the capital intensity in the good Y sector, firms with capacities higher than $\hat{\theta}$ fragment their production process and shift the production of such labor-intensive intermediate goods to the South.

Proposition 2 *If the elasticity of substitution between intermediate goods in the manufacturing is smaller than the elasticity of substitution in consumption between varieties of the manufactured good, firms with low capacities remain in the North, while firms with high capacities fragment their production process and shift the production of labor-intensive intermediate goods to the South.*

To find the industry equilibrium, let us first define the average unit costs of intermediate goods for the firm that fragments its production process as

$$\begin{aligned}\tilde{c}^F(w_N, r_N, w_S, r_S, \theta_l, \theta) &= \left[\int_{\theta_l}^{\gamma} c(w_S, r_S, i)^{1-\sigma} di + \int_{\gamma}^{\theta} c(w_N, r_N, i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\ &= \left[\tilde{c}(w_S, r_S, \theta_l, \gamma)^{1-\sigma} + \tilde{c}(w_N, r_N, \gamma, \theta)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.\end{aligned}$$

Then, the industry average of the average intermediate-goods costs can be written as

$$\begin{aligned}&\bar{c}^F(w_N, r_N, w_S, r_S) \\ &\equiv \left[\int_{\underline{\theta}}^{\hat{\theta}} \tilde{c}(w_N, r_N, \theta_l^F(\theta), \theta)^{1-\beta} h(\theta) d\theta + \int_{\hat{\theta}}^{1} \tilde{c}^F(w_N, r_N, w_S, r_S, \theta_l^F(\theta), \theta)^{1-\beta} h(\theta) d\theta\right]^{\frac{1}{1-\beta}}.\end{aligned}$$

Defining $c_k(\theta) \equiv c(w_k, r_k, \theta)$ and $\tilde{c}_k(\theta_1, \theta_2) \equiv \tilde{c}(w_k, r_k, \theta_1, \theta_2)$, for $k = S, K$, and omitting the arguments regarding the factor prices, to simplify the notation, we obtain the production level of the firm that fragment its production process as

$$X(\theta) = \frac{(\beta-1)n}{\beta m} \tilde{c}^F(\theta_l^F(\theta), \theta)^{-\beta} \left(\bar{c}^F\right)^{\beta-1}, \tag{8}$$

where θ_l^F is determined so as to satisfy

$$\frac{(\beta-1)n}{\beta m} \tilde{c}^F(\theta_l^F(\theta), \theta)^{\sigma-\beta} c_S(\theta_l^F(\theta))^{1-\sigma} \left(\bar{c}^F\right)^{\beta-1} = (\sigma-1) w f. \tag{9}$$

The total costs for fragmenting firm θ is

$$C^F(w_N, r_N, w_S, r_S, X, \theta) = \tilde{c}^F(w_N, r_N, w_S, r_S, \theta_l^F(\theta), \theta)X + [\theta - \theta_l(\theta)]wf.$$

Those for the firm that remain in the North are still the same as (6) except that θ_l is replaced by θ_l^F. These firms are now faced with the firms with different cost structure as before, so they also change the set of intermediate goods that they utilize.

Having described the industry equilibrium with some firms fragmenting their production process, we turn to the general equilibrium analysis to determine factor prices and consequently all other endogenous variables of the model. We first note that the mass of the firms in the good X industry is similarly determined except that all firms have different technology utilization than in the case where fragmentation is not an option. That is, the mass of the firms m^F is determined so as to satisfy

$$m^F = \frac{n}{\beta r_N^{\frac{\gamma}{\gamma-1}} \{f_e + f \int_{\underline{\theta}}^1 [\theta - \theta_l^F(\theta)] h(\theta) d\theta\}}. \tag{10}$$

As some firms produce part of intermediate goods in the South, the good X industry demands both labor and capital in both countries.[1] To derive the capital demands in the South by the fragmenting northern firms, we obtain

$$\frac{\partial C^F}{\partial r_S} = \tilde{c}^F(\theta_l^F(\theta), \theta)^\sigma \tilde{c}_S(\theta_l^F(\theta), \gamma)^{-\sigma} \frac{\partial \tilde{c}_S}{\partial r_S} X(\theta).$$

Then it follows from

$$\frac{\partial \tilde{c}_S}{\partial r_S} = \frac{\tilde{c}_S(\theta_l^F(\theta), \gamma)^\sigma \left[\gamma c_S(\gamma)^{1-\sigma} - \theta_l^F(\theta) c_S(\theta_l^F(\theta))^{1-\sigma} - \tilde{c}_S(\theta_l^F(\theta), \gamma)^{1-\sigma}\right]}{r(1-\sigma)\log(r/w)}$$

and

$$X(\theta) = \frac{(\beta - 1)n}{\beta m} \tilde{c}^F(\theta_l^F(\theta), \theta)^{-\beta} \left(\bar{c}^F\right)^{\beta-1}$$

[1] We treat FDI as a shift of production location from the North to the South, without accompanying capital movement across the countries.

that firm θ' demands for capital in the South can be written as

$$\frac{(\beta-1)(1-\gamma)n\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\Omega_S^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S, \theta)}{\beta m r(1-\sigma)\log r},$$

where we have used $w = r^{\frac{\gamma}{\gamma-1}}$ and

$$\begin{aligned}
&\Omega_S^F(w_N, r_N, w_S, r_S, \theta) \\
&\equiv \tilde{c}^F(w_N, r_N, w_S, r_S, \theta_l^F(\theta), \theta)^{\sigma-\beta}\left[\gamma c(w_S, r_S, \gamma)^{1-\sigma} - \theta_l^F(\theta)c(w_S, r_S, \theta_l^F(\theta))^{1-\sigma} - \tilde{c}(w_S, r_S, \theta_l^F(\theta), \gamma)^{1-\sigma}\right]
\end{aligned}$$

Similarly, fragmenting firm θ's capital demands in the North are given by

$$\frac{(\beta-1)(1-\gamma)n\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\Omega_N^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S, \theta)}{\beta m r(1-\sigma)\log r}, \tag{11}$$

where

$$\begin{aligned}
&\Omega_N^F(w_N, r_N, w_S, r_S, \theta) \\
&\equiv \tilde{c}^F(w_N, r_N, w_S, r_S, \theta_l^F(\theta), \theta)^{\sigma-\beta}\left[\theta c(w_N, r_N, \theta)^{1-\sigma} - \gamma c(w_N, r_N, \gamma)^{1-\sigma} - \tilde{c}(w_N, r_N, \gamma, \theta)^{1-\sigma}\right].
\end{aligned}$$

To find the capital market clearing condition in the South, we derive the capital demands in the good Y sector in the South. The South demands $w_S L_S + r_S K_S - n_S$ units of good Y, while it exports $n_S - A$ units of good Y to the North to finance its imports of good X, where A denotes the total rewards to the factors that are used to produce intermediate goods for fragmenting northern firms. Thus, the South produces $w_S L_S + r_S K_S - A$ units of good Y, demanding $(r_S^{\frac{\gamma}{\gamma-1}} L_S + r_S K_S - A)\gamma / r_S$ units of capital there. Now, using

$$\begin{aligned}
\int_{\theta_l^F(\theta)}^{\gamma} c_S(i)x(i)di &= \tilde{c}^F(\theta_l^F(\theta), \gamma)^{1-\sigma}X^F(\theta) \\
&= \frac{(\beta-1)n}{\beta m}\left(\bar{c}^F\right)^{\beta-1}\tilde{c}^F(\theta_l^F(\theta), \theta)^{\sigma-\beta}\tilde{c}_S(\theta_l^F(\theta), \gamma)^{1-\sigma},
\end{aligned}$$

we obtain

$$\begin{aligned} A &= \int_{\hat{\theta}}^{1} m \left[\int_{\theta_l^F(\theta)}^{\gamma} c_S(i)x(i)di \right] h(\theta)d\theta \\ &= \frac{(\beta-1)n\left(\bar{c}^F\right)^{\beta-1}}{\beta} \int_{\hat{\theta}}^{1} \tilde{c}^F(\theta_l^F(\theta),\theta)^{\sigma-\beta}\tilde{c}_S(\theta_l^F(\theta),\gamma)^{1-\sigma}h(\theta)d\theta. \end{aligned}$$

The capital market clearing condition in the South can be written as

$$\begin{aligned} &\frac{(\beta-1)(1-\gamma)n}{\beta r_S(1-\sigma)\log r_S}\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\int_{\hat{\theta}}^{1}\Omega_S^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S,\theta)h(\theta)d\theta + \frac{\gamma}{r_S}\left(r_S^{\frac{\gamma}{\gamma-1}}L_S + r_S K_S\right) \\ &- \frac{(\beta-1)\gamma n}{\beta r_S}\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\int_{\hat{\theta}}^{1}\tilde{c}^F(\theta_l^F(\theta),\theta)^{\sigma-\beta}\tilde{c}_S(\theta_l^F(\theta),\gamma)^{1-\sigma}h(\theta)d\theta = K_S, \end{aligned}$$

or equivalently,

$$\begin{aligned} &\frac{(\beta-1)(1-\gamma)n\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}}{\beta(1-\sigma)\log r_S}\int_{\hat{\theta}}^{1}\Omega_S^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S,\theta)h(\theta)d\theta + \gamma r_S^{\frac{\gamma}{\gamma-1}}L_S \\ &- \frac{(\beta-1)\gamma n}{\beta}\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\int_{\hat{\theta}}^{1}\tilde{c}^F(\theta_l^F(\theta),\theta)^{\sigma-\beta}\tilde{c}_S(\theta_l^F(\theta),\gamma)^{1-\sigma}h(\theta)d\theta \\ &= (1-\gamma)r_S K_S. \end{aligned} \tag{12}$$

We turn to the derivation of the capital market clearing condition in the North. The North demands $w_N L_N + r_N K_N - n_N$ units of good Y, while it imports $n_S - A$ units. Thus, its production of good Y equals $w_N L_N + r_N K_N - n + A$ units, demanding $(r_N^{\frac{\gamma}{\gamma-1}} L_N + r_N K_N - n + A)\gamma/r_N$ units of capital there. In the good X industry, on the other hand, firms with capacity $\theta < \hat{\theta}$ demands in aggregate

$$\frac{(\beta-1)(1-\gamma)n}{\beta(1-\sigma)\log r_N}\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\int_{\underline{\theta}}^{\hat{\theta}}\Omega_N(r_N^{\frac{\gamma}{\gamma-1}}, r_N,\theta)h(\theta)d\theta,$$

where

$$\begin{aligned} &\Omega_N(w_N, r_N, \theta) \\ &\equiv \tilde{c}(w_N, r_N, \theta_l^F(\theta),\theta)^{\sigma-\beta}\left[\theta c(w_N, r_N,\theta)^{1-\sigma} - \theta_l^F c(w_N, r_N, \theta_l^F(\theta))^{1-\sigma} - \tilde{c}(w_N, r_N, \theta_l^F(\theta),\theta)^{1-\sigma}\right]. \end{aligned}$$

We obtain also from (11) the capital market clearing condition in the North as

$$
\begin{aligned}
& \frac{(\beta-1)(1-\gamma)n}{\beta(1-\sigma)\log r_N}\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\left[\int_{\underline{\theta}}^{\hat{\theta}} \Omega_N(r_N^{\frac{\gamma}{\gamma-1}}, r_N, \theta)h(\theta)d\theta + \int_{\hat{\theta}}^{1} \Omega_N^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S, \theta)h(\theta)d\theta\right] \\
+ \;& \gamma\left(r_N^{\frac{\gamma}{\gamma-1}} L_N - n\right) + \frac{(\beta-1)\gamma n}{\beta}\bar{c}^F(r_N^{\frac{\gamma}{\gamma-1}}, r_N, r_S^{\frac{\gamma}{\gamma-1}}, r_S)^{\beta-1}\int_{\hat{\theta}}^{1} \tilde{c}^F(\theta_l^F(\theta), \theta)^{\sigma-\beta}\tilde{c}_S(\theta_l^F(\theta), \gamma)^{1-\sigma}h(\theta)d\theta \\
= \;& (1-\gamma)r_N K_N. \qquad (13)
\end{aligned}
$$

The free entry condition (10), capital market clearing conditions in the South and North, (12) and (13), determine the factor prices and other endogenous variables in the model.

5 Concluding Remarks

In the model where northern manufacturing firms choose the set of varieties of intermediate goods, we have shown that firms may not utilize all available intermediate goods and the firm with a lower capacity to utilize the intermediate goods uses only the subset of the intermediate goods that the firm with a higher capacity. When they have an option of fragmenting their production process and shifting a part to the South, firms with higher capacity tend to do so, while firms with lower capacity remain in the North, in the case where the elasticity of substitution between intermediate goods in the manufacturing is smaller than the elasticity of substitution in consumption between varieties of the manufactured good.

Our analysis focuses on the case where the higher is the capital intensity of the production of intermediate goods, the lower is the unit cost of intermediate-goods production in both countries. In reality, the opposite situation is equally likely especially in the South. In that case, firms with low capacity are also likely to produce labor-intensive intermediate goods in the South if they are given an option of FDI. We leave the extension of our model in that direction as future research.

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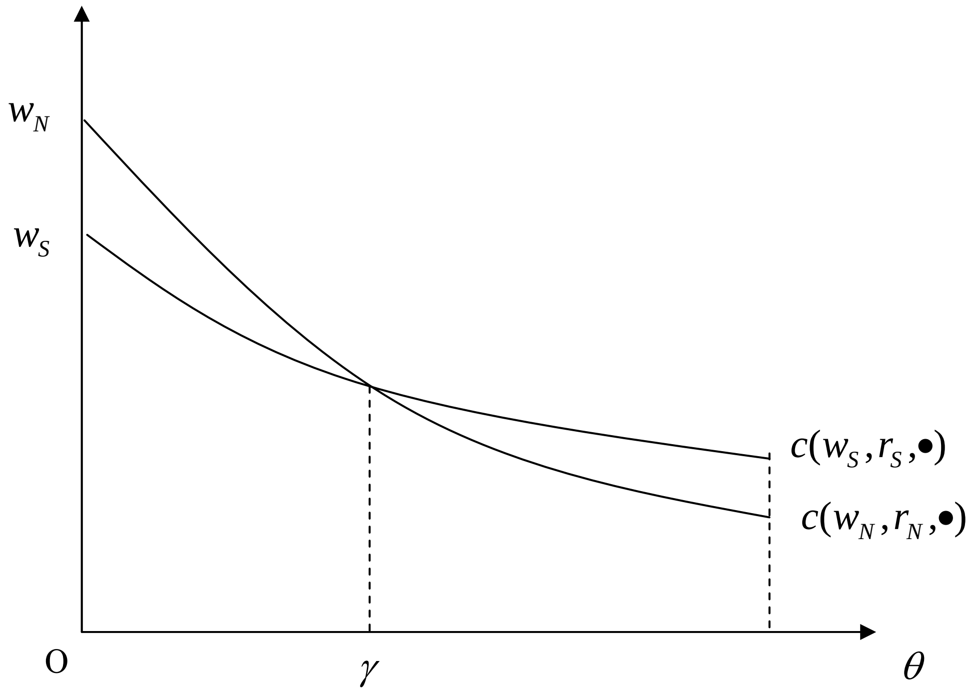

Figure 1. Unit cost functions of the intermediate goods